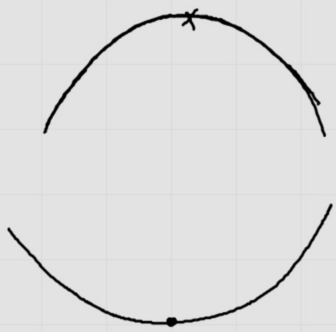
The Second Derivative Test for Relative Extrema



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If $x=c$ is a c.n and $f''(c)<0 \to f(c)$ is a rel. max.

If $x=c$ is a c.n. and $f''(c)>0 \to f(c)$ is a rel min.

SDT does not work if $f''(c.n) \nexists$ or $f''(c.n)=0$.

$f(x) = 12 + 2x^2 - x^4$ rel ext

$f'(x) = 4x - 4x^3$

$f' \exists \forall x$

$f'(x) = 0 \rightarrow x = -1$ or $x = 0$ or $x = 1$

$f''(x) = 4 - 12x^2$

f has rel max of 13 at $x = -1$ because $f''(-1) = -8 < 0 \rightarrow$ f is CD.

f has rel min of 12 at $x = 0$ because $f''(0) = 4 > 0 \rightarrow$ f C.U.

f has rel max of 13 at $x = 1$ because $f''(1) = -8 < 0 \rightarrow$ f is CD.

$f(x) = x\sqrt{x+3}$ $D_f\ [-3, \infty)$

$f'(x) = \dfrac{3x+6}{2\sqrt{x+3}}$

$f' \nexists$ when $x \leq -3$

$f'(x) = 0 \Rightarrow x = -2$

$f''(x) = \dfrac{3x+12}{4\sqrt{(x+3)^3}}$

f has rel min of -2 at $x = -2$ because

$f''(-2) = \dfrac{3}{2} > 0 \Rightarrow$ f is C.U.