

Find c.n. of $f(x) = \begin{cases} x^2 - 2x & x \leq 0 \\ 5x - 1 & x > 0 \end{cases}$.

$$f'(x) = \begin{cases} 2x - 2 & x \leq 0 \\ 5 & x > 0 \end{cases}$$

$$f'(x) \neq 0$$

$f' \nexists$ at $x=0$ because $f'_-(0) = -2$ but $f'_+(0) = 5$.

$\therefore$ C.n. is $x=0$

Find IP (if any) of $f(x) = 3\cos 2x$ on $(0, \pi)$.

$$f'(x) = -6\sin 2x$$

$$f''(x) = -12\cos 2x$$

$$f'' \neq 0 \quad \forall x \in (0, \pi)$$

$$f''(x) = 0 \rightarrow x = \frac{\pi}{4} \text{ or } x = \frac{3\pi}{4}$$

f has IP at $(\frac{\pi}{4}, 0)$ because $f''(x) < 0$ on $(0, \frac{\pi}{4})$
and $f''(x) > 0$ on $(\frac{\pi}{4}, \frac{3\pi}{4})$ and $f(\frac{\pi}{4}) = 0$.

f has IP at $(\frac{3\pi}{4}, 0)$ because $f''(x) > 0$ on $(\frac{\pi}{4}, \frac{3\pi}{4})$
and $f''(x) < 0$ on $(\frac{3\pi}{4}, \pi)$ and $f(\frac{3\pi}{4}) = 0$.

Given that $f(x) = 2x^2 + \frac{k}{x}$ has IP at $x = -1$, find k .

We know $f''(-1) = 0$.

$$f'(x) = \frac{4x^3 - k}{x^2}$$

$$f''(x) = \frac{4x^3 + 2k}{x^3}$$

$$f''(-1) = 4 - 2k$$

$$\therefore 4 - 2k = 0 \rightarrow \boxed{k = 2}$$

Given $f(x) = x^3 + 3x^2$ on $[-2, 5]$ find c guaranteed by the MVT.

$$f'(x) = 3x^2 + 6x$$

$$3x^2 + 6x = \frac{f(5) - f(-2)}{5 - (-2)}$$

$$3x^2 + 6x = 28 \rightarrow x = -4.215 \text{ or } x = 2.215$$

$$-4.215 \notin (-2, 5) \therefore c = 2.215$$

Find IP (if any) of $f(x) = \ln(1+x^2)$.

$$f'(x) = \frac{2x}{1+x^2}$$

$$f''(x) = \frac{-2x^2 + 2}{(1+x^2)^2}$$

$$f''(x) = 0 \rightarrow x = -1 \text{ or } x = 1$$

$$f'' \neq 0 \forall x$$

f has IP at $(-1, \ln 2)$ because $f''(x) < 0$ on $(-\infty, -1)$
and $f''(x) > 0$ on $(-1, 1)$ and $f(-1) = \ln 2$.

f has IP at $(1, \ln 2)$ because $f''(x) > 0$ on $(-1, 1)$
and $f''(x) < 0$ on $(1, \infty)$ and $f(1) = \ln 2$.

Find rel ext of $f(x) = x\sqrt{3-x^2}$

$$f'(x) = \frac{3-2x^2}{\sqrt{3-x^2}}$$

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-4

f' $\nexists$ on $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$

$$f'(x) = 0 \rightarrow x = -\frac{\sqrt{6}}{2} \text{ or } x = \frac{\sqrt{6}}{2}$$

$$f''(x) = \frac{2x^3 - 9x^2}{\sqrt{(3-x^2)^3}}$$

f has a rel min of $-\frac{3}{2}$ at $x = -\frac{\sqrt{6}}{2}$ because

$$f''\left(-\frac{\sqrt{6}}{2}\right) = 4 > 0 \rightarrow f \text{ is C.V.}$$

f has rel max of $\frac{3}{2}$ at $x = \frac{\sqrt{6}}{2}$ because

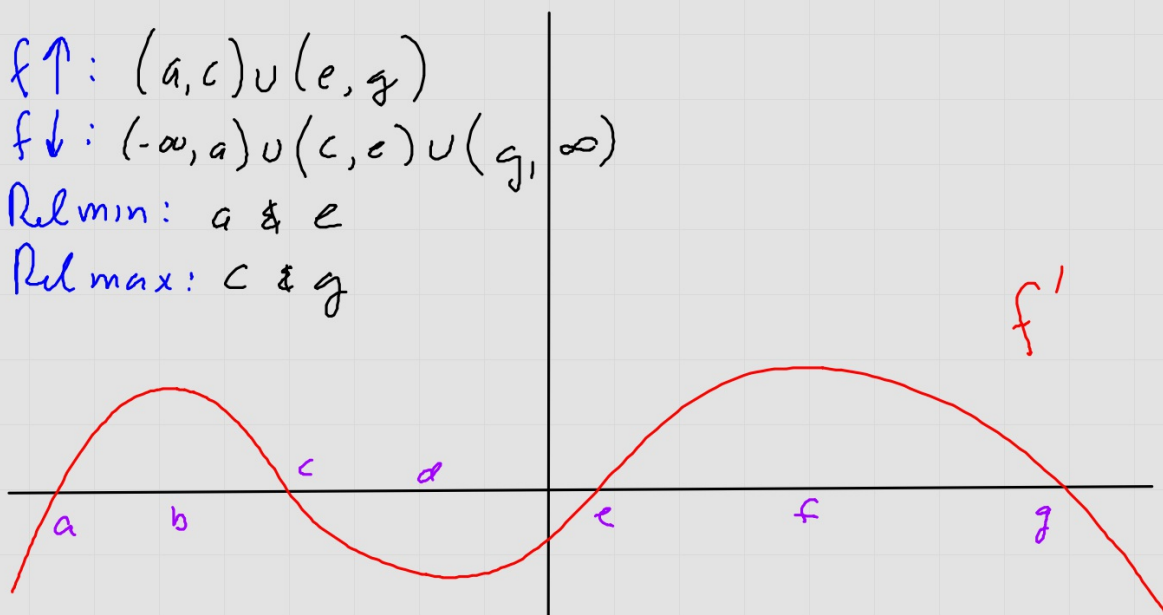
$$f''\left(\frac{\sqrt{6}}{2}\right) = -4 < 0 \rightarrow f \text{ is C.D.}$$

$$f \uparrow : (a, c) \cup (e, g)$$

$$f \downarrow : (-\infty, a) \cup (c, e) \cup (g, \infty)$$

$$\text{Rel min: } a \text{ \& } e$$

$$\text{Rel max: } c \text{ \& } g$$



$$f \subset \cup : (-\infty, b) \cup (d, f)$$

$$f \subset \cap : (b, d) \cup (f, \infty)$$

$$\text{IP} : b \text{ \& } d \text{ \& } f$$

"A" $f' > 0$ $f'' > 0$

"B" $f' > 0$ $f'' < 0$

"C" $f' < 0$ $f'' > 0$

"D" $f' < 0$ $f'' < 0$

