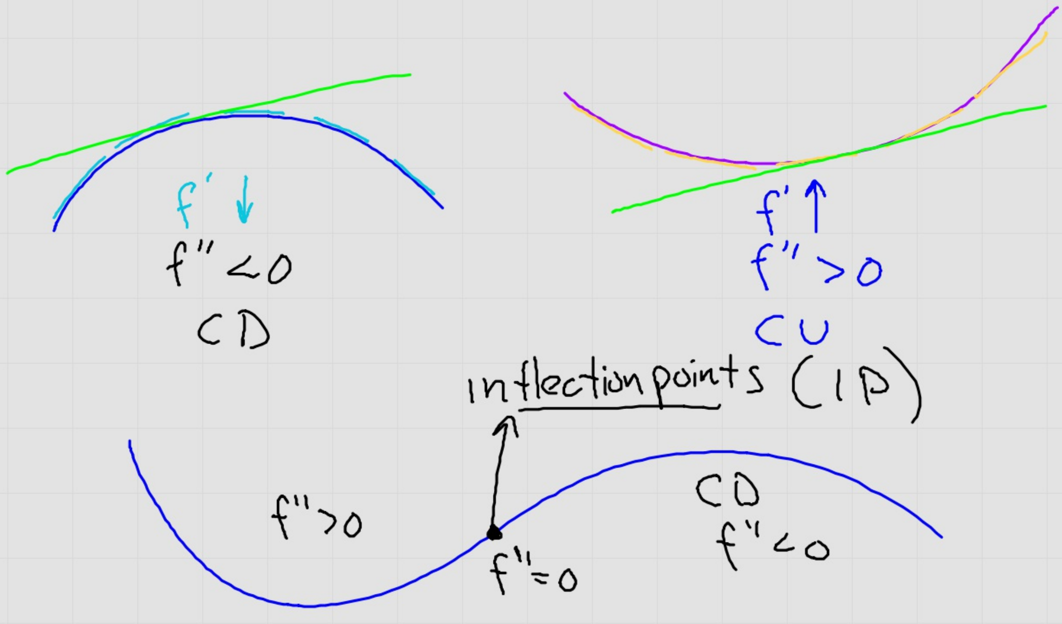
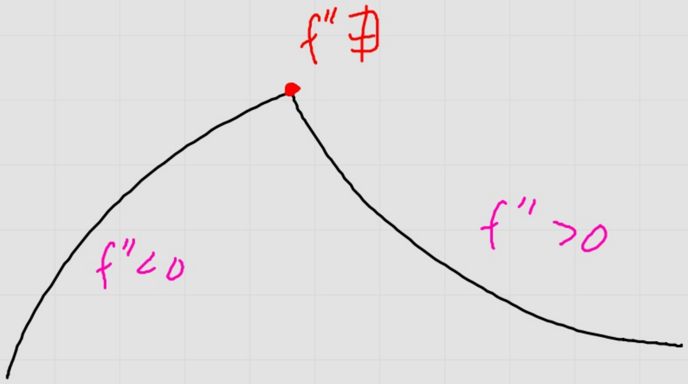
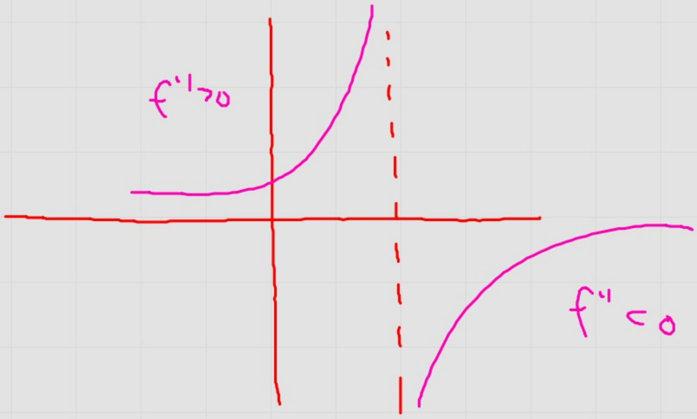
Concavity and Inflection Points

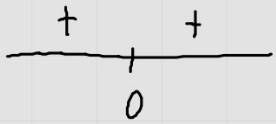




values of x where
$f''=0$ or $f'' \nexists$
"possible IP"



$f(x) = x^4$ CU/CD/IP



$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f''\ \exists\ \forall x$

$f''(x) = 0 \rightarrow x = 0$

CU/CD

f is CU on $(-\infty,0)\cup(0,\infty)$ because $f''(x)>0$ on $(-\infty,0)\cup(0,\infty)$.

IP

f has no IP because $f''(x)>0$ on $(-\infty,0)\cup(0,\infty)$.

$f(x) = \frac{x-2}{x+2}$ CD/CU/IP

$+$ | $-$ at -2

$f'(x) = \frac{4}{(x+2)^2}$

$f''(x) = -\frac{8}{(x+2)^3}$

$f''(x) \neq 0$

f'' $\nexists$ at $x=-2$

CU/CD

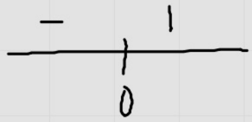
f is CU on $(-\infty,-2)$ becau $f''(x)>0$ on $(-\infty,-2)$.

f is CD on $(-2,\infty)$ becau $f''(x)<0$ on $(-2,\infty)$.

IP

f has no IP becau $f(-2)$ $\nexists$.

$f(x) = x^3 - 3x + 1$ $\cancel{CD/CU}$/IP



$f'(x) = 3x^2 - 3$

$f''(x) = 6x$

$f'' \exists \ \forall x$

$f''(x) = 0 \rightarrow x = 0$

f has IP at $(0,1)$ because $f''(x) < 0$ on $(-\infty, 0)$ and $f''(x) > 0$ on $(0, \infty)$. and $f(0) = 1$.

$f(x) = x^2 + \frac{c}{x}$ has an IP at $x = -1$, find c.

We know that $f''(-1) = 0$.

$f'(x) = 2x - cx^{-2}$

$f''(x) = \frac{2x^3 + 2c}{x^3}$

$f''(-1) = 2 - 2c$

$\therefore 2 - 2c = 0 \longrightarrow \boxed{c = 1}$

Inc / Dec / Rel Ext

$f' = 0$ or $f' \nexists$ c.n.

$f' > 0$ $f \uparrow$

$f' < 0$ $f \downarrow$

Test all c.n.

f' $+ \to -$ rel max

f' $- \to +$ rel min

CU / CD / IP

$f'' = 0$ or $f'' \nexists$ PIP

$f'' > 0$ f CU

$f'' < 0$ f CD

Test all PIP

f'' $+ \to -$ or $- \to +$

and f $\exists$ there

IP