Find the c.n. of $f(x) = \begin{cases} x^2 - 2x & x \leq 0 \\ 5x - 1 & x > 0. \end{cases}$

$$f'(x) = \begin{cases} 2x - 2 & x < 0 \\ 5 & x > 0 \end{cases}$$

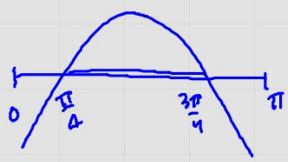
$f'(x) \neq 0$

$f' \nexists$ at $x = 0$ because $f'_-(0) = -2$ but $f'_+(0) = 5$.

$\therefore$ c.n. is $x = 0$ only.

Find the IP (if any) of $f(x) = 3\cos 2x$ on $(0, \pi)$.

$f'(x) = -6\sin 2x$



$f''(x) = -12\cos 2x$

$f'' \ \exists \ \forall x \in (0, \pi)$

$f''(x) = 0 \rightarrow x = \frac{\pi}{4}$ & $x = \frac{3\pi}{4}$

f has IP at $\left(\frac{\pi}{4}, 0\right)$ because $f''(x) < 0$ on $\left(0, \frac{\pi}{4}\right)$ and $f''(x) > 0$ on $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ and $f\left(\frac{\pi}{4}\right) = 0$.

f has an IP at $\left(\frac{3\pi}{4}, 0\right)$ because $f''(x) > 0$ on $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ and $f''(x) < 0$ on $\left(\frac{3\pi}{4}, \pi\right)$ and $f\left(\frac{3\pi}{4}\right) = 0$.

Given $f(x) = 2x^2 + \frac{k}{x}$ has IP at $x = -1$, find k.

We know $f''(-1) = 0$.

$$f'(x) = \frac{4x^3 - k}{x^2}$$

$$f''(x) = \frac{4x^3 + 2k}{x^3}$$

$$f''(-1) = 4 - 2k$$

$$\therefore 4 - 2k = 0 \rightarrow k = 2.$$

Given $f(x) = x^3 + 3x^2$ on $[-2, 5]$ find the c guaranteed by the MVT.

$$3x^2 + 6x = \frac{f(5) - f(-2)}{5 - -2}$$

$$3x^2 + 6x = 28 \longrightarrow x = -4.215 \text{ or } x = 2.215$$

$$-4.215 \notin (-2, 5) \quad \therefore \; c = 2.215$$

Find IP (if any) of $f(x) = \ln(1+x^2)$.

$$f'(x) = \frac{2x}{1+x^2}$$

$$f''(x) = \frac{-2x^2+2}{(1+x^2)^2}$$

$f'' \exists \forall x$

$f''(x) = 0 \Rightarrow x = -1$ or $x = 1$

f has IP at $(-1, \ln 2)$ because $f''(x) < 0$ on $(-\infty, -1)$ and $f''(x) > 0$ on $(-1, 1)$ and $f(-1) = \ln 2$.

f has IP at $(1, \ln 2)$ because $f''(x) > 0$ on $(-1, 1)$ and $f''(x) < 0$ on $(1, \infty)$ and $f(1) = \ln 2$.

Find rel ext of $f(x) = x\sqrt{3-x^2}$.

$$f'(x) = \frac{3-2x^2}{\sqrt{3-x^2}}$$

$f' \exists$ only on $(-\sqrt{3}, \sqrt{3})$

4
-4
$f'(x) = 0 \to x = -\frac{\sqrt{6}}{2}$ or $x = \frac{\sqrt{6}}{2}$.

$$f''(x) = \frac{2x^3 - 9x}{\sqrt{(3-x^2)^3}}$$

f has rel min of $-\frac{3}{2}$ at $x = -\frac{\sqrt{6}}{2}$ because
$f''\left(-\frac{\sqrt{6}}{2}\right) = 4 > 0 \to f$ is C.U.

f has rel max of $\frac{3}{2}$ at $= \frac{\sqrt{6}}{2}$ because
$f''\left(\frac{\sqrt{6}}{2}\right) = -4 < 0 \to f$ is C.D.

$f \uparrow$: $(a,c) \cup (e,g)$

$f \downarrow$: $(-\infty,a) \cup (c,e) \cup (g,\infty)$

Rel max: c & g

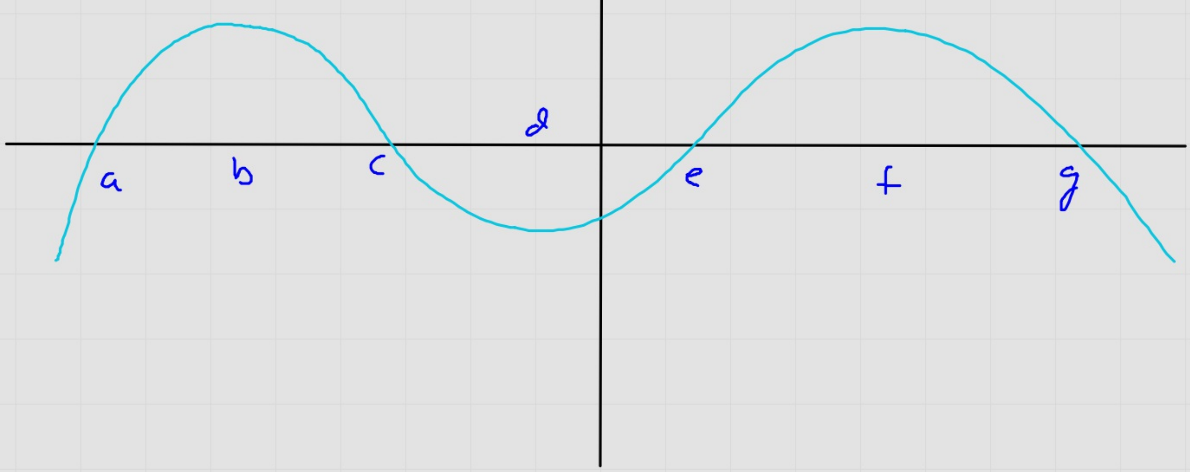
Rel min: a & e

f CU: $(-\infty,a) \cup (d,f)$

f CD: $(b,d) \cup (f,\infty)$

f IP: b & d & f

f'



"A" $f' > 0$ $f'' > 0$

"B" $f' > 0$ $f'' < 0$

"C" $f' < 0$ $f'' > 0$

"D" $f' < 0$ $f'' < 0$