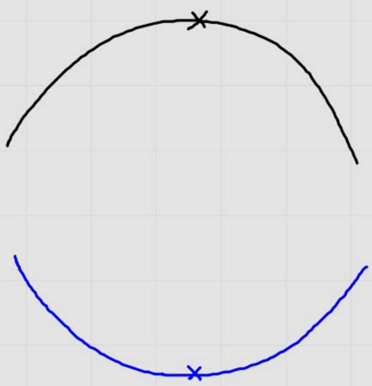
The Second Derivative Test for Relative Extrema



If $x=c$ is a c.n and $f''(c)<0 \rightarrow f(c)$ is a rel. max.

If $x=c$ is a c.n and $f''(c)>0 \rightarrow f(c)$ is a rel min.

* If $f''(c.n)=0$ or $f''(c.n) \nexists$ can't use SDT.

$f(x) = 12 + 2x^2 - x^4$ rel.ext.

$f'(x) = 4x - 4x^3$

$f' \exists \forall x$

$f'(x) = 0 \Rightarrow x = -1 \vee x = 0 \vee x = 1$

$f''(x) = 4 - 12x^2$

f has a rel max of 13 at $x = -1$ because

$f''(-1) = -8 < 0 \rightarrow f$ C.D.

f has a rel min of 12 at $x = 0$ because

$f''(0) = 4 > 0 \rightarrow f$ CU.

f has a rel max of 13 at $x = 1$ because $f''(1) = -8 < 0 \rightarrow$ CD.

$f(x) = x\sqrt{x+3}$ $\qquad$ $D: [-3, \infty)$

$$f'(x) = \frac{3x+6}{2\sqrt{x+3}}$$

$f' \nexists$ when $x \leq -3$.

$f'(x) = 0 \Rightarrow x = -2$

$$f''(x) = \frac{3x+12}{4\sqrt{(x+3)^3}}$$

f has a rel min of -2 at $x = -2$

because $f''(-2) = \frac{3}{2} > 0 \Rightarrow$ f CU.