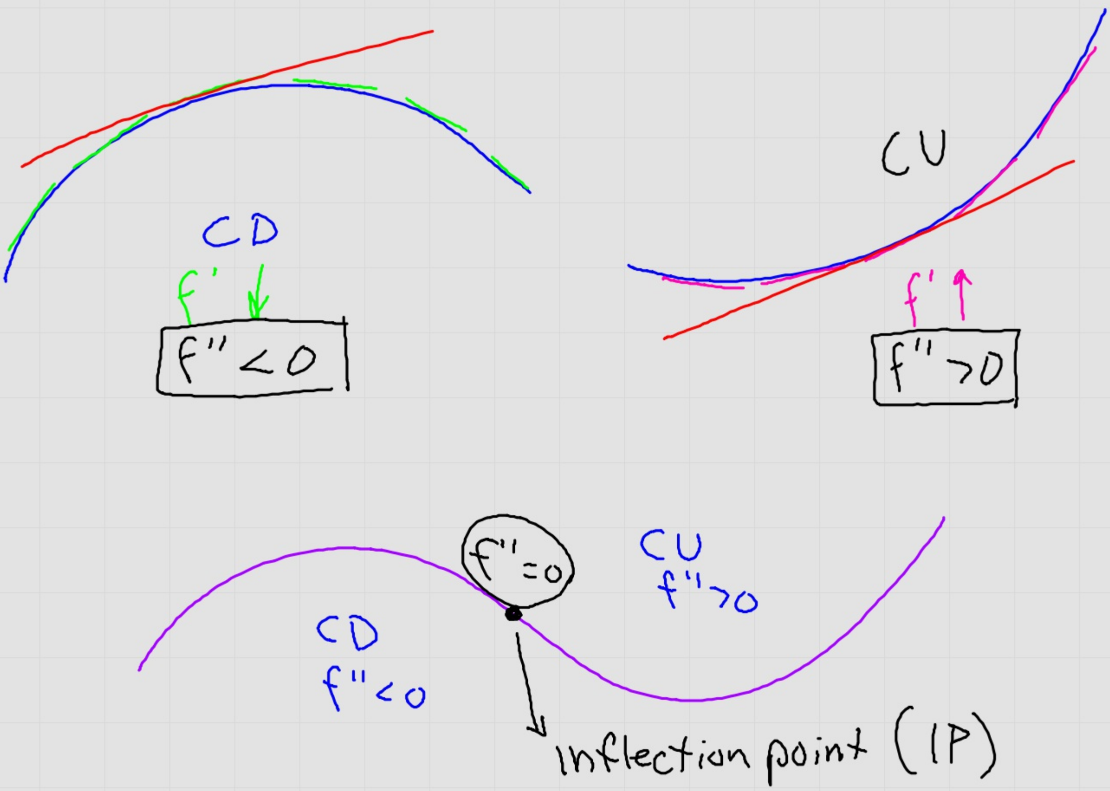
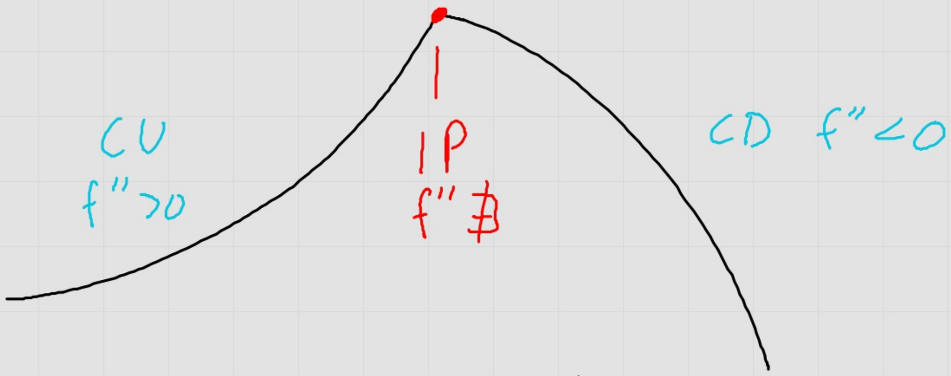
Concavity and Inflection Points

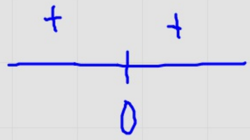




Values of x where f'' ∄ or $f'' = 0$

"possible IP"

$f(x) = x^4 \quad CU/CD\ /\ IP$



$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f''\ \exists\ \forall x$

$f''(x) = 0 \Rightarrow x = 0$

$f'' = 0$ but no I.P.

CU/CD

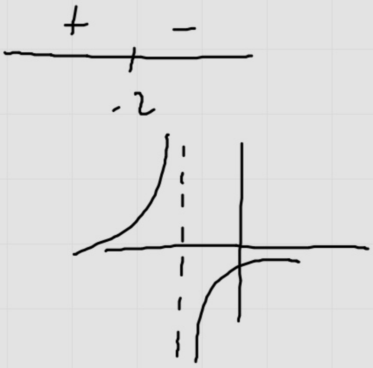
f is CU on $(-\infty, 0) \cup (0, \infty)$
because $f''(x) > 0$ on $(-\infty, 0) \cup (0, \infty)$.

IP

f has no IP.

$f(x) = \frac{x-1}{x+2}$ CU/CD/IP

$f'(x) = \frac{3}{(x+2)^2}$



$f''(x) = -\frac{6}{(x+2)^3}$

$f''(x) \neq 0$

f'' $\nexists$ at $x = -2$

<u>CU/CD</u>

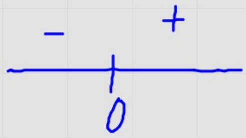
f is CU on $(-\infty, -2)$ because $f''(x) > 0$ on $(-\infty, -2)$.

f is CD on $(-2, \infty)$ because $f''(x) < 0$ on $(-2, \infty)$.

<u>IP</u>

f has no IP because $f''(x) > 0$ on $(-\infty, -2)$ and $f''(x) < 0$ on $(-2, \infty)$ <u>but</u> $f(-2)$ $\nexists$.

$f(x) = x^3 - 3x + 1$ CD/CU/IP



$f'(x) = 3x^2 - 3$

$f''(x) = 6x$

$f''\ \exists\ \forall x$

$f''(x) = 0 \Rightarrow x = 0$

CU/CD

f is CD on $(-\infty, 0)$ because $f''(x) < 0$ on $(-\infty, 0)$.

f is CU on $(0, \infty)$ because $f''(x) > 0$ on $(0, \infty)$.

IP

f has an IP at $(0, 1)$ because $f''(x) < 0$ on $(-\infty, 0)$ and $f''(x) > 0$ on $(0, \infty)$ and $f(0) = 1$.

$f(x) = x^2 + \frac{c}{x}$ has an IP at $x = -1$, find c.

We know that $f''(-1) = 0$.

$$f'(x) = \frac{2x^3 - c}{x^2} \quad \text{and} \quad f''(x) = \frac{2x^3 + 2c}{x^3}$$

$$f''(-1) = 2 - 2c$$

$$\therefore 2 - 2c = 0$$

$$c = 1$$

Incr / Decr / Rel Ext

$f' > 0$ $f \uparrow$

$f' < 0$ $f \downarrow$

$f' = 0$ or $f' \nexists$ c.n.

Test all c.n.

f' $+ \to -$ rel max

f' $- \to +$ rel min

CD / CU / IP

$f'' > 0$ CU

$f'' < 0$ CD

$f'' = 0$ or $f'' \nexists$ PIP

Test all PIP

f'' $+ \to -$ OR $- \to +$ IP