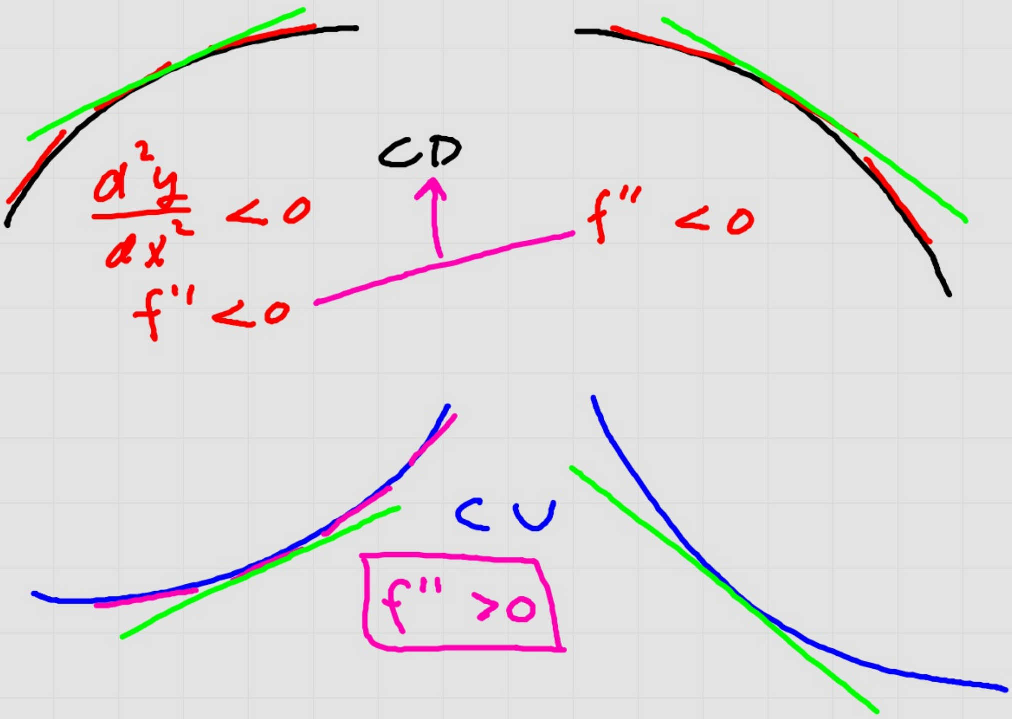
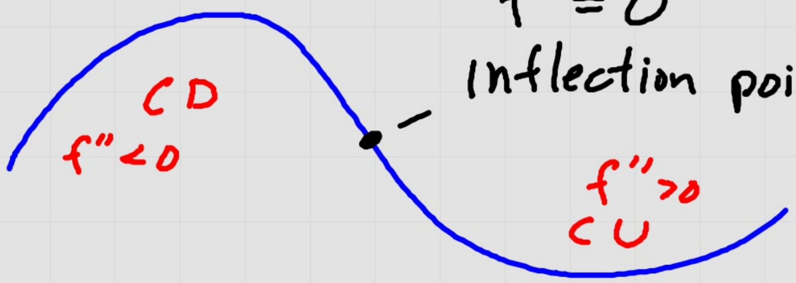
Concavity and Inflection Points



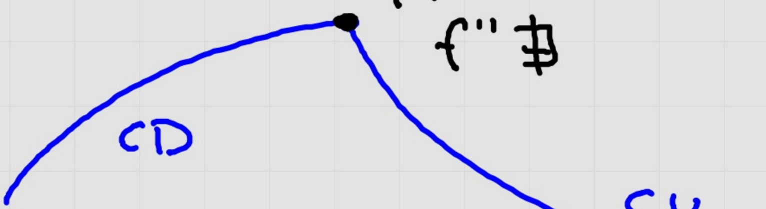
$f'' = 0$

Inflection point (IP)



IP

$f'' \nexists$



$f'' = 0$ or $f'' \nexists$ "possible IP"

Just because $f''=0$ or $f''\nexists$ does not necessarily mean we have IP.

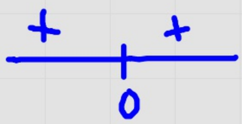
$f(x)=x^4$

$f'(x)=4x^3$

$f''(x)=12x^2$

$f''\ \exists\ \forall x$

$f''(x)=0 \rightarrow x=0$



$$f(x) = \frac{x-3}{x-2}$$

$$f'(x) = \frac{1}{(x-2)^2}$$

$$f''(x) = -\frac{2}{(x-2)^3}$$

$$f''(x) \neq 0$$

f'' $\nexists$ at $x = 2$

<u>CU/CD</u>

f is CU on $(-\infty, 2)$ because $f''(x) > 0$ on $(-\infty, 2)$.

f is CD on $(2, \infty)$ because $f''(x) < 0$ on $(2, \infty)$.

$$\begin{array}{ccc} + & & - \\ \hline & 2 & \end{array}$$

<u>IP</u>

$f(2)$ $\nexists$ $\therefore$ no IP

$f(x) = x^3 - 3x + 1 \quad$ CU/CD? IP?

$f'(x) = 3x^2 - 3$

$f''(x) = 6x$

$f'' \ \exists \ \forall x$

$f''(x) = 0 \rightarrow x = 0$

$$\begin{array}{ccc} - & & + \\ \hline & 0 & \end{array}$$

<u>CD/CU</u>

f is CD on $(-\infty, 0)$ because $f''(x) < 0$ on $(-\infty, 0)$.

f is CU on $(0, \infty)$ because $f''(x) > 0$ on $(0, \infty)$.

<u>IP</u>

f has IP at $(0,1)$ because $f''(x) < 0$ on $(-\infty, 0)$ and $f''(x) > 0$ on $(0, \infty)$ and $f(0) = 1$.

$x^2 + cx^{-1}$

Given that $f(x) = x^2 + \frac{c}{x}$ has an IP at $x = -1$, find c.

We know that $f''(-1) = 0$.

$$f'(x) = 2x - cx^{-2}$$

$$f''(x) = 2 + 2cx^{-3}$$

$$= 2 + \frac{2c}{x^3} = \frac{2x^3 + 2c}{x^3}$$

$$f''(-1) = \frac{-2 + 2c}{-1} = 2 - 2c$$

$$\therefore 2 - 2c = 0$$

$$c = 1$$

Inc/Dec/Rel Ext

$$\left.\begin{array}{l} f' = 0 \\ f' \nexists \end{array}\right\} \text{c.n.}$$

$f' > 0$ $f \uparrow$

$f' < 0$ $f \downarrow$

Test for rel ext

f' $+ \to -$ rel max

f' $- \to +$ rel min

CD/CU/IP

$$\left.\begin{array}{l} f'' = 0 \\ f'' \nexists \end{array}\right\} \text{PIP}$$

$f'' > 0$ CU

$f'' < 0$ CD

Test for IP

f'' $+ \to -$ or $- \to +$

and $f(\text{PIP}) \exists$

then IP.