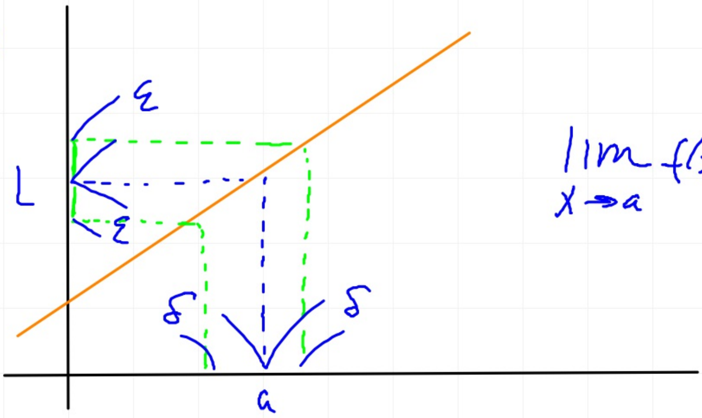
The Definition of Limit

ε epsilon

δ delta

$\exists$ such that



$$\lim_{x \to a} f(x) = L.$$

$\lim_{x \to a} f(x) = L$ is true if $\forall \varepsilon > 0 \; \exists \delta > 0 \; \ni$

when $|x - a| < \delta \longrightarrow |f(x) - L| < \varepsilon$

Given $\lim_{x \to 3} (2x-1) = 5$ and $\varepsilon = .01$ find an appropriat δ.

$$2x_1 - 1 = 5.01 \qquad 2x_2 - 1 = 4.99$$
$$2x_1 = 6.01 \qquad 2x_2 = 5.99$$
$$x_1 = 3.005 \qquad x_2 = 2.995$$
$$\delta_1 = .005 \qquad \delta_2 = .005$$

$\therefore$ choose $\delta \leq .005$

Given $\lim_{x \to 3} (2x-1) = 5$ and $\varepsilon = .01$ find an appro. δ.

For $\varepsilon = .01$ we need find a $\delta > 0 \ni$

when $|x-3| < \delta \rightarrow |(2x-1)-5| < .01$

$$|2x-6| < .01$$

$$|x-3| < \frac{.01}{2}$$

$\therefore$ choose $\delta \leq \frac{.01}{2}$.

Prove: $\lim_{x \to 3} (2x-1) = 5.$

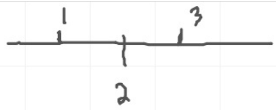
Pf: We need to show that $\forall \varepsilon > 0 \; \exists \delta > 0 \ni$ when $|x-3| < \delta \to |(2x-1)-5| < \varepsilon$

$$|2x-6| < \varepsilon$$

$$|x-3| < \frac{\varepsilon}{2}$$

$\therefore$ Choose $\delta = \min\{1, \frac{\varepsilon}{2}\}$

Q.E.D.



Prove: $\lim_{x \to 2} (x^2 + 2x + 6) = 14$.

Pf: We need to show that $\forall \varepsilon > 0\ \exists \delta > 0 \ni$ when $|x-2| < \delta \rightarrow |(x^2+2x+6) - 14| < \varepsilon$

$$|x^2+2x-8| < \varepsilon$$

$$|x-2|\,|x+4| < \varepsilon$$

$$|x-2| < \frac{\varepsilon}{|x+4|}$$

Consider $[1,3]$

$x = 1 \rightarrow \frac{\varepsilon}{|x+4|} = \frac{\varepsilon}{5}$

$x = 3 \rightarrow \frac{\varepsilon}{|x+4|} = \frac{\varepsilon}{7}$ $\quad\therefore$ Choose $\delta = \min\left\{1, \frac{\varepsilon}{7}\right\}$.