Limits of Trigonometric Functions

But first...a quick review of domains and the greatest integer function

Find the domain of $f(x) = \dfrac{x+3}{x^2+5x+6}$.

$f \nexists$ when $x^2+5x+6=0 \longrightarrow x=-3$ or $x=-2$.

$\therefore$ the domain of f is $(-\infty,-3) \cup (-3,-2) \cup (-2,\infty)$.

Find the domain of $f(x) = \sqrt{x-2}$.

$f \exists$ when $x-2 \geq 0$

$\therefore$ the domain of f is $[2, \infty)$.

Find the domain of $f(x) = \sqrt{x^2+5x+6}$.

$f \exists$ when $x^2+5x+6 \geq 0$.

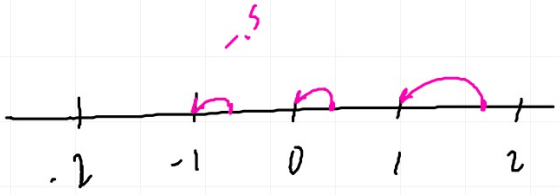
$x^2+5x+6 = 0 \rightarrow x = -3$ or $x = -2$

$\therefore$ the domain of f is $(-\infty, -3] \cup [-2, \infty)$.

$f(x) = ||x||$ greatest integer func.

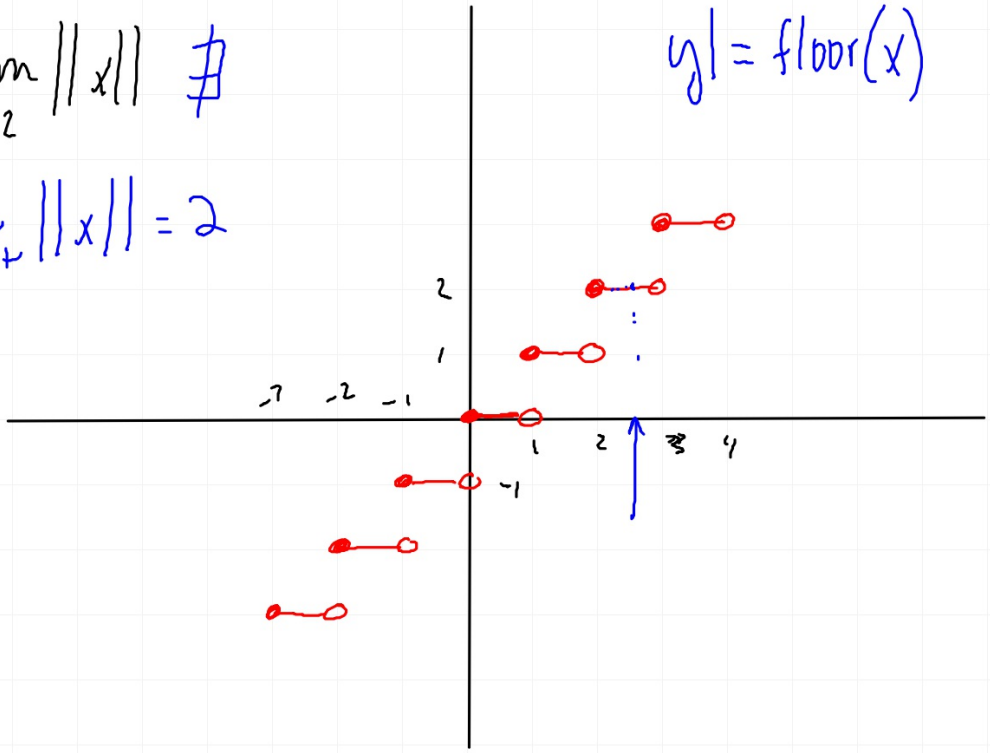
$f(0) = 0$
$f(.5) = 0$
$f(.9) = 0$
$f(1) = 1$
$f(2.001) = 2$
$f(2.6) = 2$
$f(3) = 3$

$f(-1.6) = -2$



$\lim_{x \to 2} \|x\| \not\exists$

$\lim_{x \to 2^+} \|x\| = 2$



$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{3x \to 0} \frac{\sin 3x}{3x} = 1$$

$$\frac{\sin \bigstar}{\bigstar} \to 1 \qquad \frac{1 - \cos \bigstar}{\bigstar} \to 0$$

$$\lim_{x \to 0} \frac{\sin 7x}{2x} = \lim_{x \to 0} \frac{\frac{\sin 7x}{7x} \cdot 7x}{2x} = \frac{7}{2}$$

$$\lim_{x \to 0} \frac{\sin 9x}{\sin 3x} = \lim_{x \to 0} \frac{\frac{\sin 9x}{9x} \cdot 9x}{\frac{\sin 3x}{3x} \cdot 3x} = 3$$

$$\lim_{x \to 0} \frac{5x}{\sin 2x} = \lim_{x \to 0} \frac{5x}{\frac{\sin 2x}{2x} \cdot 2x} = \frac{5}{2}$$

$$\lim_{x \to 0} \frac{x+7}{\cos x} = 7$$

$$\lim_{x\to 0} \frac{\tan x}{2x} = \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{2x} = \lim_{x\to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{2x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{2\cos x}$$

$$= \frac{1}{2}.$$

$$\lim_{x\to 0} \frac{\sin^2 3x}{5x^2} = \lim_{x\to 0} \frac{\sin 3x \cdot \sin 3x}{5x^2}$$

$$= \lim_{x\to 0} \frac{\frac{\sin 3x}{3x} \cdot \frac{\sin 3x}{3x} \cdot 9x^2}{5x^2} = \frac{9}{5}.$$