Limit Theorems

(How we really find limits!)

Plug in the "a"

Result of plugging in	Limit
c	c
$\frac{0}{0}$	not done $\to$ factor, rationalize, L'Hopital
$\frac{c}{0}$	$\nexists$ L & R $\pm\infty$

Ex Eval $\lim\limits_{x \to 2} \frac{x^2 - 4}{x - 2}$

$$\lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = 3$$

OR

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 3$$

Ex Eval $\lim_{x \to 5} \frac{x-7}{x-5}$

$$\lim_{x \to 5} \frac{x-7}{x-5} \nexists$$

$$\lim_{x \to 5^+} \frac{x-7}{x-5} = -\infty \quad \text{and} \quad \lim_{x \to 5^-} \frac{x-7}{x-5} = +\infty$$

(Ex) Find the VA (if any) of $f(x) = \frac{x-7}{x-5}$.

f has a VA at $x=5$

because $f(5) \nexists$ and $\lim_{x \to 5} f(x) = \pm\infty$.

Ex Eval $\lim_{h \to 0} \frac{4 - \sqrt{16-h}}{h}$

$$\lim_{h \to 0} \frac{4 - \sqrt{16-h}}{h} \cdot \frac{4 + \sqrt{16-h}}{4 + \sqrt{16-h}}$$

$$= \lim_{h \to 0} \frac{16 - (16-h)}{h\left(4 + \sqrt{16-h}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\,1}{\cancel{h}\left(4 + \sqrt{16+h}\right)}$$

$$= \frac{1}{8}$$

Ex Eval $\lim_{x \to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x - 2}$

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} \cdot \frac{2x}{2x}$$

$$= \lim_{x \to 2} \frac{\overset{-1}{\cancel{2 - x}}}{2x\cancel{(x-2)}}$$

$$= -\frac{1}{4}$$

Ex Given $f(x) = \begin{cases} \sqrt{x-4} & x > 4 \\ 8-2x & x < 4 \end{cases}$ find $\lim_{x \to 4} f(x)$.

$\lim_{x \to 4} f(x) = 0$ because $\lim_{x \to 4^+} f(x) = 0$

and $\lim_{x \to 4^-} f(x) = 0$

$g(x) = \begin{cases} \sqrt{x+4} & x > 4 \\ 8-2x & x < 4 \end{cases}$ $\lim_{x \to 4} g(x)$.

$\lim_{x \to 4} g(x) \nexists$ because $\lim_{x \to 4^+} g(x) = \sqrt{8}$

but $\lim_{x \to 4^-} g(x) = 0$.

(Ex) Eval $\lim_{x \to 7} |x-7|$.

$$|x-7| = \begin{cases} x-7 & x \geq 7 \\ 7-x & x < 7 \end{cases}$$

$\lim_{x \to 7} |x-7| = 0$ because

$\lim_{x \to 7^+} |x-7| = 0$ and $\lim_{x \to 7^-} |x-7| = 0$.

Ex Given $h(x) = \frac{|x-2|}{x-2}$ find $\lim_{x \to 2} h(x)$.

$$h(x) = \begin{cases} \frac{x-2}{x-2} & x > 2 \\ \frac{2-x}{x-2} & x < 2 \end{cases} = \begin{cases} 1 & x > 2 \\ -1 & x < 2 \end{cases}$$

$\lim_{x \to 2} h(x) \nexists$ because $\lim_{x \to 2^+} h(x) = 1$

but $\lim_{x \to 2^-} h(x) = -1$.