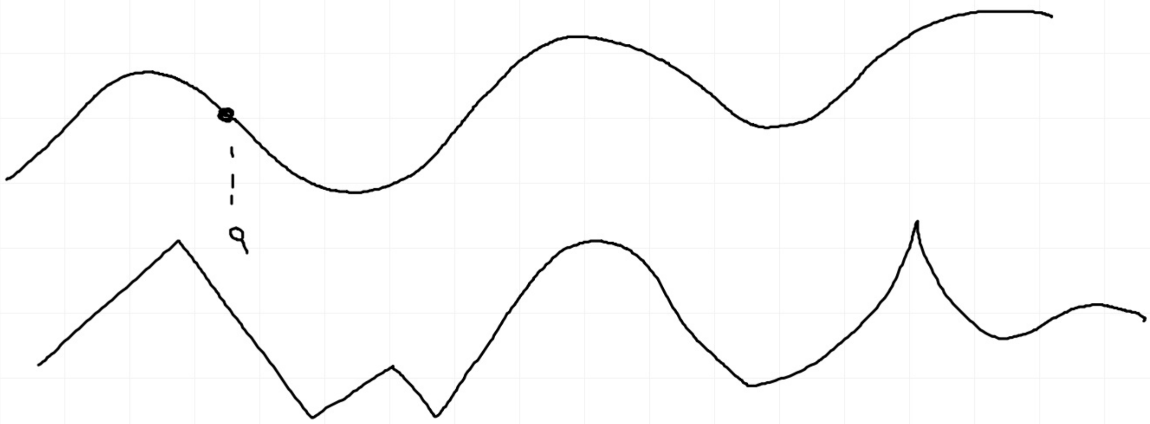
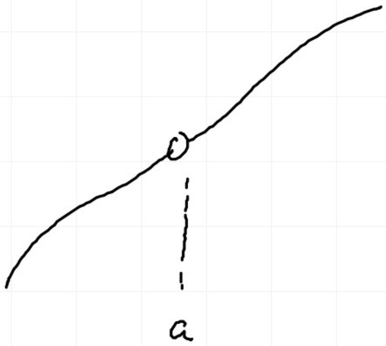
Continuity

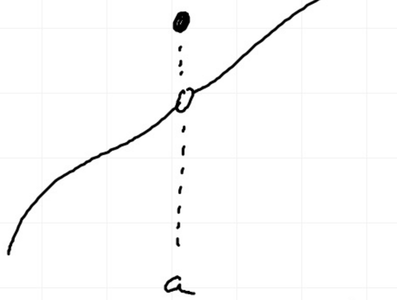
at a number

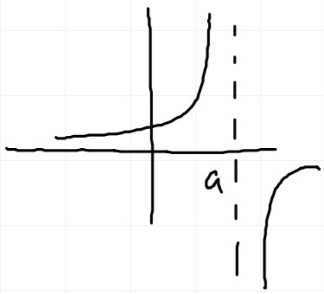
on an interval



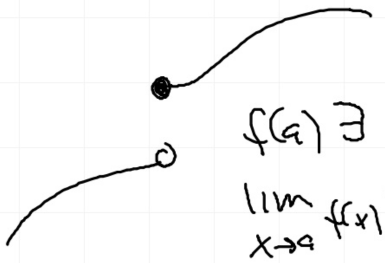


$f(a)\ \nexists$



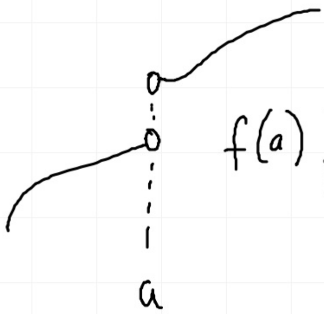


$f(a)\ \exists \qquad \lim_{x \to a} f(x)\ \exists$



$f(a)\ \exists$

$\lim_{x \to a} f(x)\ \nexists$



$f(a)\ \nexists$

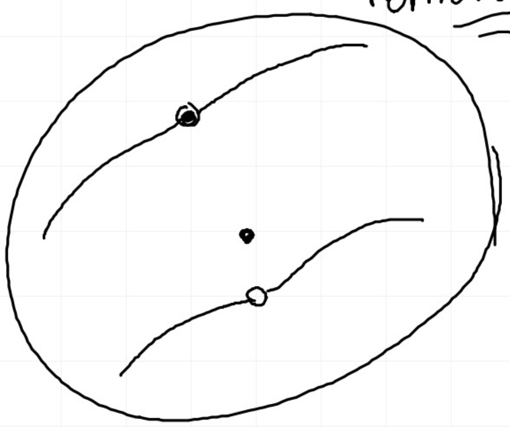
$\underline{f \text{ is cont at } x=a \text{ iff}}$

$f(a)\ \exists$

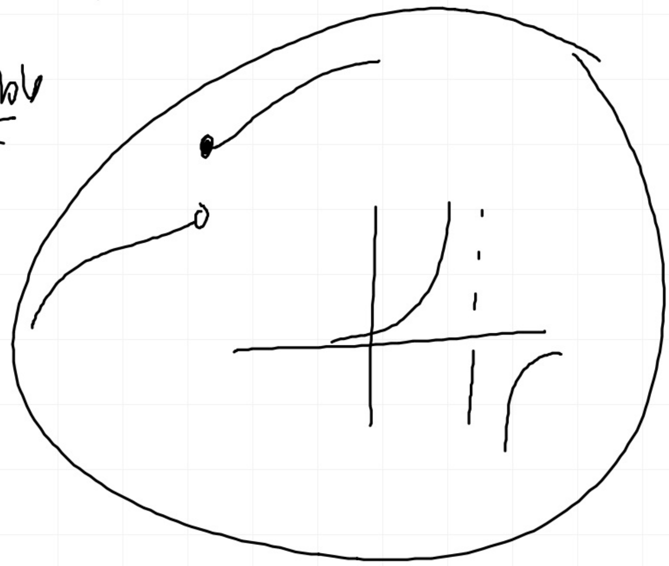
$\lim_{x \to a} f(x)\ \exists$

$f(a) = \lim_{x \to a} f(x)$

removable



essential



Removable

$\Big\downarrow$

if $\lim\limits_{x \to a} f(x) \; \exists$

Essential

$\Big\downarrow$

if $\lim\limits_{x \to a} f(x) \; \nexists$.

Is $f(x) = \dfrac{x^2+2x-15}{x-3}$ cont at $x=3$?

<u>Cont test at $x=3$</u>

$f(3) \nexists$ $\therefore$ f is

not cont at $x=3$.

Is $f(x) = \dfrac{x+3}{x-7}$ cont at $x=7$?

<u>Cont test at $x=7$</u>

$f(7) \nexists \therefore f$ discon at $x=7$.

Is $f(x) = \begin{cases} x^2 & x < 2 \\ 2x & x \geq 2 \end{cases}$ cont at $x=2$?

Cont test at $x=2$

$f(2) = 4$

$\left.\begin{array}{l} \lim_{x \to 2^+} f(x) = 4 \\ \lim_{x \to 2^-} f(x) = 4 \end{array}\right\} \therefore \lim_{x \to 2} f(x) = 4$

f is cont at $x=2$ because $f(2) = \lim_{x \to 2} f(x)$.

Is $f(x) = \begin{cases} x^2 & x \le 3 \\ x-5 & x > 3 \end{cases}$ cont at $x=3$?

<u>Cont test at $x=3$</u>

$f(3) = 9$

$$\left.\begin{array}{l} \lim\limits_{x \to 3^+} f(x) = -2 \\ \lim\limits_{x \to 3^-} f(x) = 9 \end{array}\right\} \therefore \lim_{x \to 3} f(x) \nexists.$$

f is discont. at $x=3$ because $\lim\limits_{x \to 3} f(x) \nexists$.

Determine where (if anywhere) $f(x)=\begin{cases} x^2-49 & x\le -7 \\ 49-x^2 & -7<x<7 \\ 5-x & x\ge 7\end{cases}$

is discon.

Cont test at $x=-7$

$f(-7)=0$

$\left.\begin{array}{l}\lim\limits_{x\to -7^+} f(x)=0 \\ \lim\limits_{x\to -7^-} f(x)=0\end{array}\right\} \therefore \lim\limits_{x\to -7} f(x)=0$

f is cont at $x=-7$ because

$f(-7)=\lim\limits_{x\to -7} f(x)$.

Cont test at $x=7$

$f(7)=-2$

$\left.\begin{array}{l}\lim\limits_{x\to 7^+} f(x)=-2 \\ \lim\limits_{x\to 7^-} f(x)=0\end{array}\right\} \therefore \lim\limits_{x\to 7} f(x) \nexists$

$\therefore$ f discon. at $x=7$

because $\lim\limits_{x\to 7} f(x) \nexists$.

For what value(s) of b will $f(x) = \begin{cases} \frac{x^2-25}{x-5} & x \neq 5 \\ b & x = 5 \end{cases}$

be cont at $x=5$?

<u>Cont test at $x=5$</u>

$f(5) = b$

$\lim_{x \to 5} f(x) = 10$

$\therefore$ for f to be cont at

$x=5 \rightarrow b=10.$

Determine if $f(x) = \dfrac{x^2+2x-15}{x-3}$ is cont at $x=3$.
If discon determine if discon. is removable or essen.
If removable, redefine f so that it is cont at $x=3$.

<u>Cont test at $x=3$</u>

f is discon at $x=3$ because $f(3) \nexists$.

<u>Removable/Essent.</u>

The disc. is removable because $\lim_{x \to 3} f(x) = 8$.

<u>Redefine f</u>

$$f(x) = \begin{cases} \dfrac{x^2+2x-15}{x-3} & x \neq 3 \\ 8 & x=3 \end{cases}$$

$$f(x) = \frac{x+2}{x-3}$$

Cont on $(0,5)$ no

$(-2,3)$ yes

$[3,5]$ no