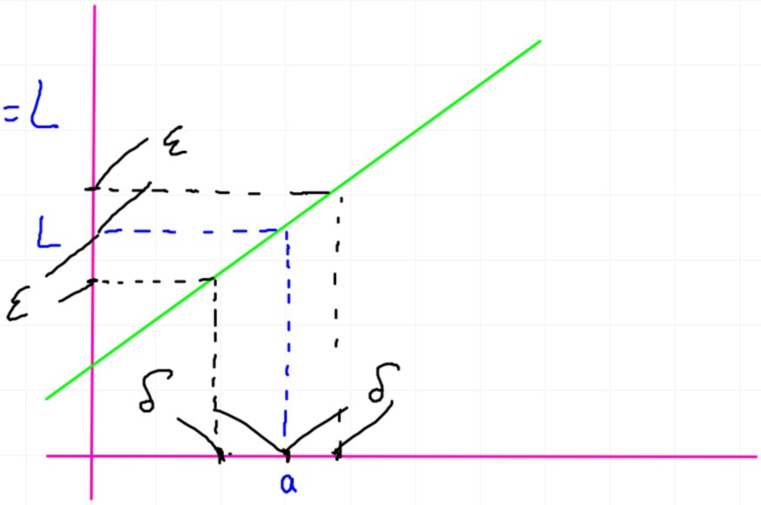
The Definition of Limit

ε epsilon

δ delta

$$\lim_{x \to a} f(x) = L$$

$\ni$ such that



$$\lim_{x \to a} f(x) = L \text{ is true if } \forall \varepsilon > 0 \ \exists \delta > 0 \ni$$

$$\text{when } |x-a| < \delta \rightarrow |f(x) - L| < \varepsilon$$

Given $\lim_{x \to 2} (3x-1) = 5$ and $\varepsilon = .01$ find an appropriate δ.

$3x_1 - 1 = 5.01$

$3x_1 = 6.01$

$x_1 = 2.003$

$\delta_1 = .003$

$3x_2 - 1 = 4.99$

$3x_2 = 5.99$

$x_2 = 1.997$

$\delta_2 = .003$

$\therefore$ choose $\delta \leq .003$

Given $\lim_{x \to 2} (3x-1) = 5$ and $\varepsilon = .01$ find an appro. δ.

For $\varepsilon = .01$ we need to find a $\delta > 0 \ni$
when $|x-2| < \delta \rightarrow |(3x-1)-5| < .01$

$$|3x-6| < .01$$

$$|x-2| < \frac{.01}{3}$$

$\therefore$ Choose $\delta \leq \frac{.01}{3}$

Prove: $\lim_{x \to 2} (3x-1) = 5.$

Pf: We need to show that $\forall \varepsilon > 0 \ \exists \ \delta > 0 \ni$ when $|x-2| < \delta \to |(3x-1)-5| < \varepsilon$

$$|3x-6| < \varepsilon$$

$$|x-2| < \frac{\varepsilon}{3}$$

$\therefore$ choose $\delta = \min\{1, \frac{\varepsilon}{3}\}$

Given $\lim_{x \to 3} (x^2 + 2x - 1) = 14$ and $\varepsilon = .001$, find an appropriate δ.

For $\varepsilon = .001$ we need to find a $\delta > 0 \ni$ when $|x-3| < \delta \rightarrow |(x^2+2x-1)-14| < .001$

$$|x^2+2x-15| < .001$$

$$|x-3||x+5| < .001$$

$$|x-3| < \frac{.001}{|x+5|}$$

Consider $[2,4]$

$x = 2 \rightarrow \frac{.001}{|x+5|} = \frac{.001}{7}$

$x = 4 \rightarrow \frac{.001}{|x+5|} = \frac{.001}{9}$

$\therefore$ choose $\delta \leq \frac{.001}{9}$

Prove $\lim_{x \to 3} (x^2+2x-1) = 14$

Pf: We need to show that $\forall \varepsilon > 0 \ \exists \delta > 0 \ni$
when $|x-3| < \delta \longrightarrow |(x^2+2x-1)-14| < \varepsilon$

$$|x^2+2x-15| < \varepsilon$$
$$|x-3|\,|x+5| < \varepsilon$$
$$|x-3| < \frac{\varepsilon}{|x+5|}$$

Consider $[2,4]$

$x=2 \rightarrow \frac{\varepsilon}{|x+5|} = \frac{\varepsilon}{7}$

$x=4 \rightarrow \frac{\varepsilon}{|x+5|} = \frac{\varepsilon}{9}$

$\therefore$ choose $\delta = \min\left\{1, \frac{\varepsilon}{9}\right\}$.