Limits of Trigonometric Functions

But first...a quick review of domains and the greatest integer function

Find the domain of $f(x) = \dfrac{x-2}{x^2+5x+6}$.

$f \nexists$ when $x^2+5x+6=0 \longrightarrow x=-3$ or $x=-2$.

$\therefore$ the domain of f is $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$.

Find the domain of $g(x) = \sqrt{x-4}$.

$g \exists$ when $x-4 \geq 0$

$\therefore$ the domain of g is $[4,\infty)$

Find the domain of $h(x) = \sqrt{x^2+5x+6}$.

$h \ \exists$ when $x^2+5x+6 \geq 0$.

$x^2+5x+6=0 \longrightarrow x=-3$ or $x=-2$

$\therefore$ the domain of h is $(-\infty,-3] \cup [-2,\infty)$.

$f(x) = ||x||$

$f(0) = 0$

$f(.5) = 0$

$f(.9) = 0$

$f(.9999) = 0$

$f(1) = 1$

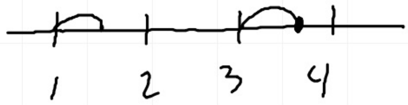
$f(1.7) = 1$

$f(1.9) = 1$

$f(2) = 2$

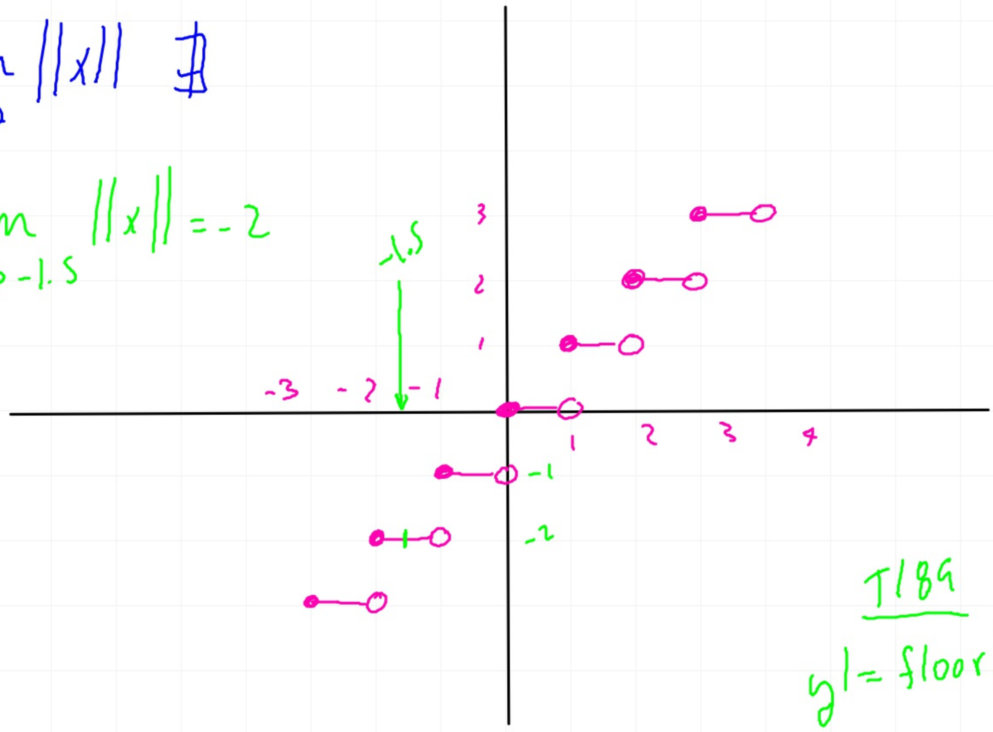
$f(7.6) = 7$

$f(-1.2) = -2$



$$\lim_{x \to 2} [\![x]\!] \ \nexists$$

$$\lim_{x \to -1.5} [\![x]\!] = -2$$



TI89

$y1 = \text{floor}(x)$

$$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1} \qquad \boxed{\lim_{x \to 0} \frac{1-\cos x}{x} = 0}$$

$$\frac{\sin \star}{\star} \to 1 \qquad \frac{1-\cos \star}{\star} \to 0$$

$$\lim_{x \to 0} \frac{\sin 5x}{3x} = \lim_{x \to 0} \frac{\overset{1}{\frac{\sin 5x}{5x}} \cdot 5\cancel{x}}{3\cancel{x}} = \frac{5}{3}$$

$$\lim_{x \to 0} \frac{8x}{\sin 3x} = \lim_{x \to 0} \frac{8x}{\frac{\sin 3x}{3x} \cdot 3x} = \frac{8}{3}$$

$$\lim_{x \to 0} \frac{\sin 7x}{\sin 5x} = \lim_{x \to 0} \frac{\frac{\sin 7x}{7x} \cdot 7x}{\frac{\sin 5x}{5x} \cdot 5x} = \frac{7}{5}.$$

$$\lim_{x \to 1} \frac{1 - \cos 8x}{3x} = \lim_{x \to 0} \frac{\overset{0}{\frac{1-\cos 8x}{8x}} \cdot 8x}{3x} = 0$$

$$\lim_{x\to 0}\frac{x^2}{\sin^2 3x} = \lim_{x\to 0}\frac{\overset{1}{x^2}}{\frac{\sin 3x}{3x}\frac{\sin 3x}{3x}\cdot 9x^2} = \frac{1}{9}$$

$$\lim_{x\to 0}\frac{\tan x}{2x} = \lim_{x\to 0}\frac{\sin x}{\cos x}\cdot\frac{1}{2x} = \lim_{x\to 0}\overset{1}{\frac{\sin x}{x}}\cdot\frac{1}{2\cos x}$$

$$= \frac{1}{2}.$$

$$\frac{\frac{\sin x}{\cos x}}{2x} = \frac{\sin x}{\cos x}\cdot\frac{1}{2x}$$

$$= \frac{\sin x}{x}\,\frac{1}{2\cos x}$$