Limit Theorems

(How we really find limits!)

$$\lim_{x \to 2} (x^2+1) = 5$$

Finding a limit as $x \to a$

Plug in the "a"

Result of	Limit
c	c
$\frac{0}{0}$	not done → factor / rationalize / L'Hopital's Rule
$\frac{c}{0}$	$\nexists$ L & R $\pm\infty$

E1 $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$$

OR

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4$$

E2

$$\lim_{x \to 2} (x^2 + 1)$$

$$\lim_{x \to 2} (x^2 + 1) = 5$$

EX 3

$$\lim_{x \to 4} \frac{x+2}{x-4}$$

$$\lim_{x \to 4} \frac{x+2}{x-4} \nexists .$$

$$\lim_{x \to 4^+} \frac{\overset{+}{x+2}}{\underset{+}{x-4}} = +\infty \text{ and } \lim_{x \to 4^-} \frac{\overset{+}{x+2}}{\underset{-}{x-4}} = -\infty$$

Ex 4 Find the VA (if any) of $f(x) = \dfrac{x+2}{x-4}$.

f has a VA at $x = 4$ because

$f(4) \nexists$ and $\lim_{x \to 4} f(x) = \pm\infty$.

EX 5 $\displaystyle\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3} = \lim_{x \to 9} \frac{\cancel{x - 9}}{\cancel{(x - 9)}(\sqrt{x} + 3)}$$

$$= \frac{1}{6}$$

$\boxed{\text{EX 6}}$ $\lim_{x \to 1} \frac{x^2 - x - 2}{x + 1}$

$$\lim_{x \to 1} \frac{x^2 - x - 2}{x + 1} = -1$$

EX7 $\displaystyle\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2}$

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2} \cdot \frac{2x}{2x} = \lim_{x \to 2} \frac{\overset{-1}{\cancel{2-x}}}{\cancel{(x-2)}\,2x} = -\frac{1}{4}.$$

Ex 8 Given $f(x) = \begin{cases} \sqrt{x-3} & x > 4 \\ 8-2x & x < 4 \end{cases}$ find $\lim_{x\to 4} f(x)$.

$\lim_{x\to 4} f(x) \nexists$ because $\lim_{x\to 4^+} f(x) = 1$

but $\lim_{x\to 4^-} f(x) = 0$.

Ex 9 $\lim\limits_{x \to 7} |x-7|$

$$|x-7| = \begin{cases} x-7 & x \geq 7 \\ 7-x & x < 7 \end{cases}$$

$\lim\limits_{x \to 7} |x-7| = 0$ because

$\lim\limits_{x \to 7^+} |x-7| = 0$ and $\lim\limits_{x \to 7^-} |x-7| = 0$.

Ex 10 Given $f(x) = \dfrac{|x-2|}{x-2}$ find $\lim_{x \to 2} f(x)$.

$$f(x) = \begin{cases} \dfrac{x-2}{x-2} & x > 2 \\ \dfrac{2-x}{x-2} & x < 2 \end{cases} = \begin{cases} 1 & x > 2 \\ -1 & x < 2 \end{cases}$$

$\lim_{x \to 2} f(x) \nexists$ because $\lim_{x \to 2^+} f(x) = 1$

but $\lim_{x \to 2^-} f(x) = -1$