The Limit of a Function

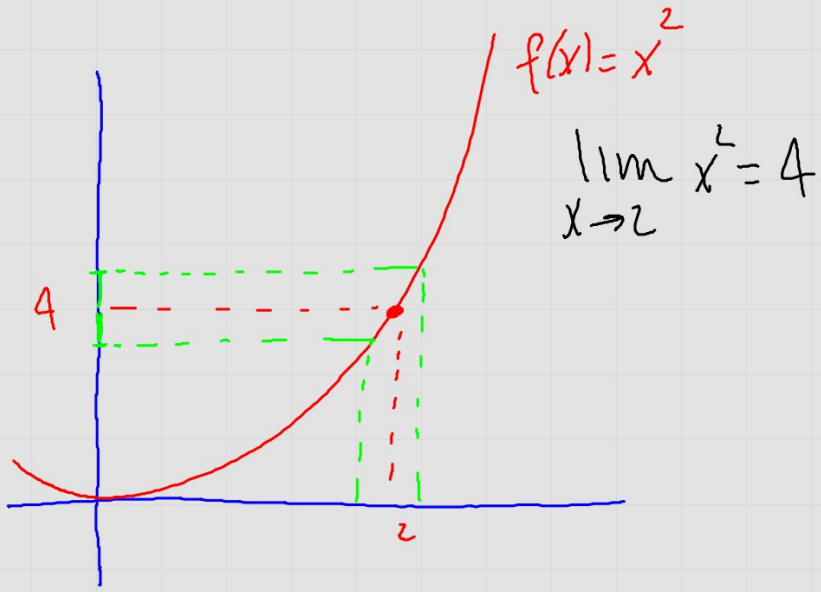
- finding a limit from a graph
- estimating limits using tables
- vertical asymptotes

$x \to a$

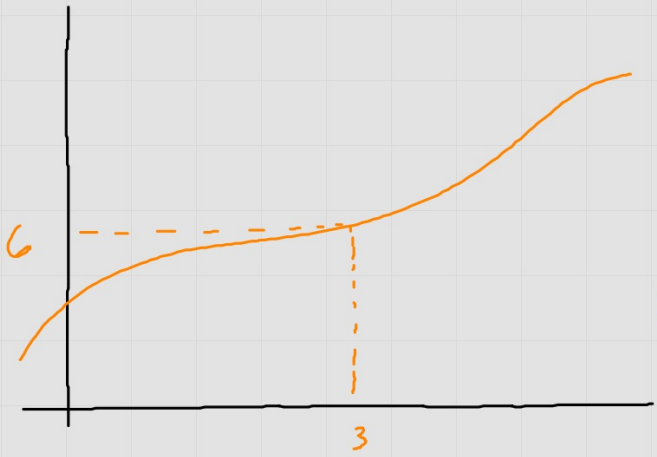
$$\lim_{x \to 5} \frac{x^2 - 25}{x - 5} = 10$$

$x \to \infty$
or
$x \to -\infty$

$\lim_{x \to a} f(x) = L$ is true if we can generate function values as close to L as we want by using inputs sufficiently close to (but not equal to) $x = a$.



$$\lim_{x \to 2} x^2 = 4$$



$f(3) = 6$

$\lim_{x \to 3} f(x)$

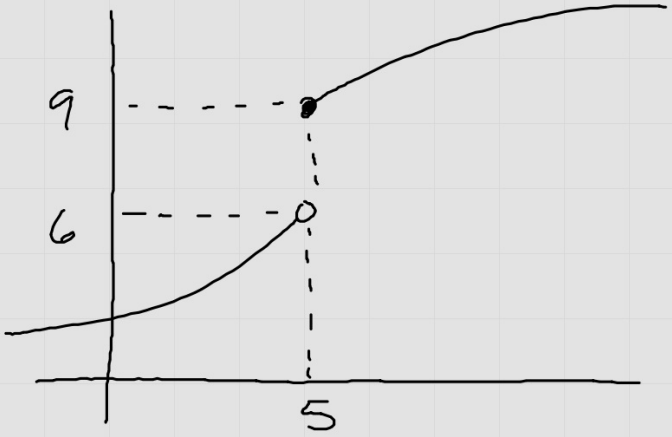
$\lim_{x \to 3} f(x) = 6$ because $\lim_{x \to 3^+} f(x) = 6$ and $\lim_{x \to 3^-} f(x) = 6$.

$f(5) \nexists$

$\lim_{x \to 5} f(x) = 8$ because $\lim_{x \to 5^+} f(x) = 8$ and $\lim_{x \to 5^-} f(x) = 8$

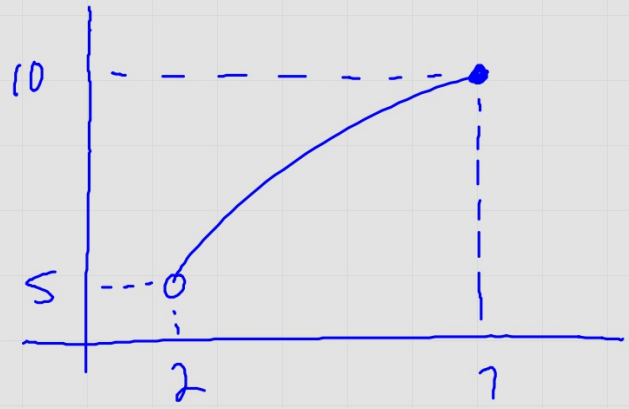
$f(7) = 87$

$\lim_{x \to 7} f(x) = 3$

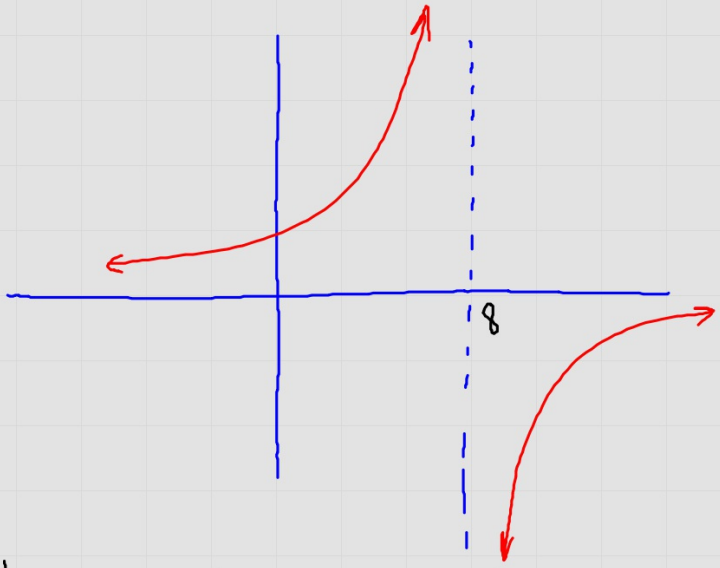


$\lim_{x \to 5} f(x)$ ∄ because $\lim_{x \to 5^+} f(x) = 9$ but

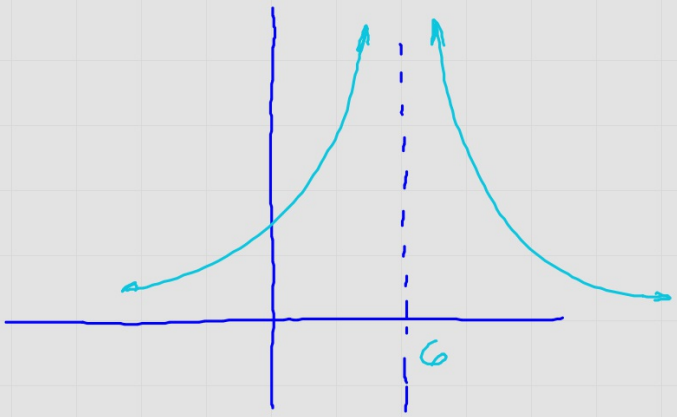
$\lim_{x \to 5^-} f(x) = 6$.



$$\lim_{x \to 2^+} f(x) = 5$$



$\lim_{x \to 8^-} f(x) = +\infty$ $\lim_{x \to 8^+} f(x) = -\infty$



$\lim_{x \to 6} f(x) \nexists$

$\lim_{x \to 6} f(x) = \pm\infty$

$\lim_{x \to 6^-} f(x) = +\infty$

$\lim_{x \to 6^+} f(x) = +\infty$

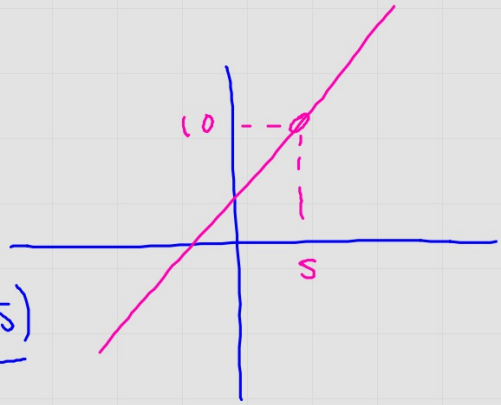
VA

f has a VA at $x=a$ iff

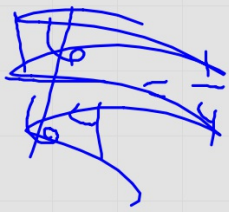
$f(a) \nexists$ and $\lim_{x \to a} f(x) = \pm\infty$

$$f(x) = \frac{x^2 - 25}{x - 5}$$

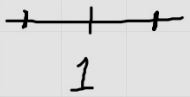
$$g(x) = \frac{1}{x - 3}$$



$$\frac{(x-5)(x+5)}{x-5}$$



Est $\lim_{x \to 1} \frac{x-1}{x^2-1}$



x	$\frac{x-1}{x^2-1}$
1.500	.400
1.100	.476
1.010	.498
1.001	.500

x	$\frac{x-1}{x^2-1}$
.500	.667
.900	.526
.990	.503
.999	.500

$\lim_{x \to 1} \frac{x-1}{x^2-1} = \frac{1}{2}$ because $\lim_{x \to 1^+} \frac{x-1}{x^2-1} = \frac{1}{2}$

and $\lim_{x \to 1^-} \frac{x-1}{x^2-1} = \frac{1}{2}$.

Est. $\lim_{x \to 3} \frac{1}{x-3}$

L

x	$\frac{1}{x-3}$
2.500	-2
2.900	-10
2.990	-100
2.999	-1000

R

x	$\frac{1}{x-3}$
3.500	2
3.100	10
3.010	100
3.001	1000

$\lim_{x \to 3} \frac{1}{x-3}$ DNE.

$\lim_{x \to 3^+} \frac{1}{x-3} = +\infty$ and $\lim_{x \to 3^-} \frac{1}{x-3} = -\infty$.

$$\lim_{x \to 0} \sin \frac{\pi}{x}$$

x	$\sin \frac{\pi}{x}$
.500	
.100	
.010	
.001	
.0001	