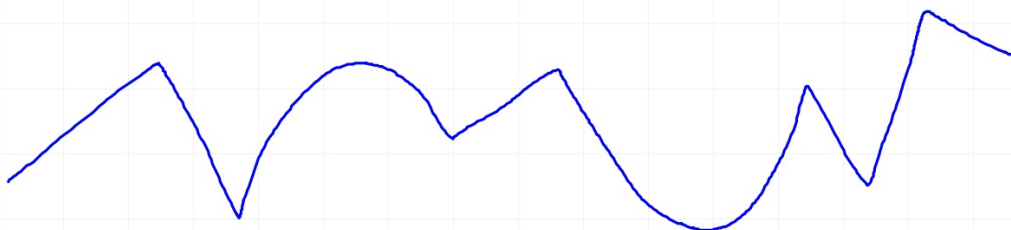
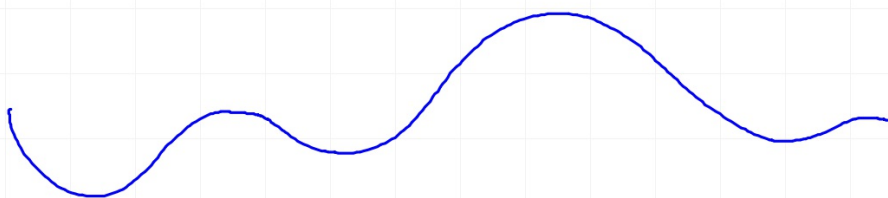
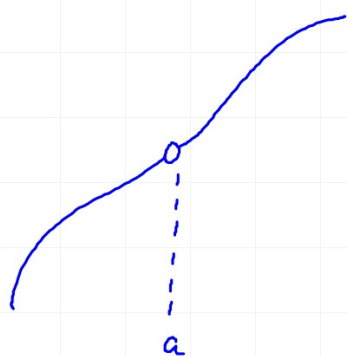


Continuity

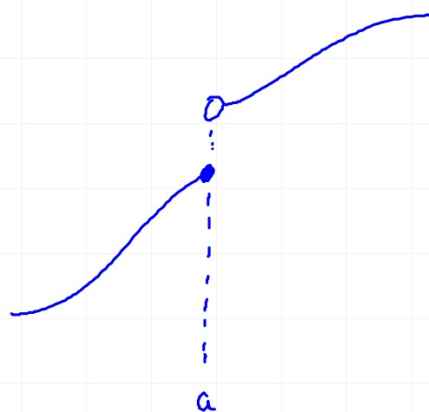
↙
at a number

↘
on an interval



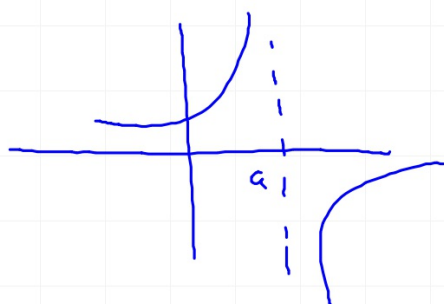
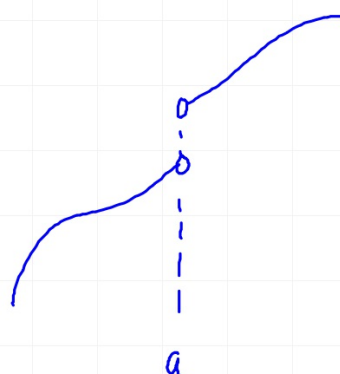


$f(a) \nexists$



$f(a) \exists$

$\lim_{x \rightarrow a} f(x) \nexists$



f is cont at $x=a$ iff

$$f(a) \exists$$

$$\lim_{x \rightarrow a} f(x) \exists$$

$$f(a) = \lim_{x \rightarrow a} f(x)$$

$$Is \ f(x) = \frac{x^2 - x - 2}{x - 2} \text{ cont at } x = 2?$$

Cont test at $x = 2$

$$f(2) \nexists \therefore f \text{ is disc. at } x = 2.$$

$$Is \ g(x) = \frac{x + 3}{x - 7} \text{ cont at } x = 7?$$

Cont test at $x = 7$

$$g(7) \nexists \therefore g \text{ is disc. at } x = 7.$$

Removable vs Essential discon.



if $\lim_{x \rightarrow a} f(x) \exists$



$\lim_{x \rightarrow a} f(x) \nexists$

Is $f(x) = \frac{x^2 + 2x - 15}{x - 3}$ cont at $x = 3$?



Is $f(x) = \frac{x^2 + 2x - 15}{x - 3}$ cont at $x = 3$? Determine if the disc. is removable or essential. If removable, redefine f so that it is cont.

Cont test at $x = 3$

$f(3) \nexists \therefore f$ is
discon. at $x = 3$.

Remov. or Essen

Discon. is removable
because $\lim_{x \rightarrow 3} f(x) = 8$.

Redefine f

$$f(x) = \begin{cases} \frac{x^2 + 2x - 15}{x - 3} & x \neq 3 \\ 8 & x = 3 \end{cases}$$

$$\text{Is } f(x) = \begin{cases} x^2 & x < 2 \\ 2x & x \geq 2 \end{cases} \text{ cont at } x=2?$$

Cont test at $x=2$

$$f(2) = 4$$

$$\left. \begin{array}{l} \lim_{x \rightarrow 2^+} f(x) = 4 \\ \lim_{x \rightarrow 2^-} f(x) = 4 \end{array} \right\} \therefore \lim_{x \rightarrow 2} f(x) = 4.$$

$\therefore f$ is cont at $x=2$ because $f(2) = \lim_{x \rightarrow 2} f(x)$.

Determine where $f(x) = \begin{cases} x^2 - 49 & x \leq -7 \\ 49 - x^2 & -7 < x < 7 \\ 5 - x & x \geq 7 \end{cases}$ is discontinuous.

Cont test at $x = 7$

$$f(7) = -2$$

$$\left. \begin{array}{l} \lim_{x \rightarrow 7^+} f(x) = -2 \\ \lim_{x \rightarrow 7^-} f(x) = 0 \end{array} \right\} \therefore \lim_{x \rightarrow 7} f(x) \nexists$$

f is discontinuous at $x = 7$

because $\lim_{x \rightarrow 7} f(x) \nexists$.

Cont test at $x = -7$

$$f(-7) = 0$$

$$\left. \begin{array}{l} \lim_{x \rightarrow -7^+} f(x) = 0 \\ \lim_{x \rightarrow -7^-} f(x) = 0 \end{array} \right\} \therefore \lim_{x \rightarrow -7} f(x) = 0$$

$\therefore f$ is continuous at $x = -7$

because $f(-7) = \lim_{x \rightarrow -7} f(x)$.

For what value(s) of b will $f(x) = \begin{cases} \frac{x^2 - 25}{x - 5} & x \neq 5 \\ b & x = 5 \end{cases}$
be cont at $x = 5$.

Cont test at $x = 5$

$$f(5) = b$$

$$\lim_{x \rightarrow 5} f(x) = 10$$

$\therefore$ For f to be cont at $x = 5$, $b = 10$.

$$f(x) = \frac{x-3}{x-5}$$

Cont on $[5, 7)$ no

$(5, 7)$ yes

$(-7, 0)$ yes

$(-3, 3)$ ~~no~~ yes

$[-1, 10)$ no