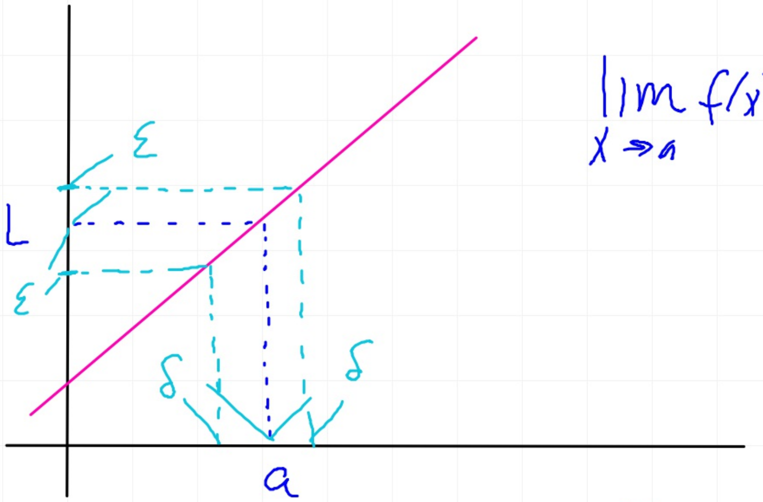
The Definition of Limit

ε - epsilon

δ - delta

$\ni$ "such that"



$$\lim_{x \to a} f(x) = L$$

$\lim_{x \to a} f(x) = L$ is true if $\forall \varepsilon > 0 \ \exists \delta > 0 \ \ni$

whenever $|x - a| < \delta \rightarrow |f(x) - L| < \varepsilon$.

Given $\lim_{x \to 2} (3x-1) = 5$ and $\varepsilon = .01$ find an appropriate δ.

$$3x_1 - 1 = 5.01 \qquad 3x_2 - 1 = 4.99$$

$$3x_1 = 6.01 \qquad 3x_2 = 5.99$$

$$x_1 = 2.005 \qquad x_2 = 1.996$$

$$\delta_1 = .005 \qquad \delta_2 = .004$$

$\therefore$ choose $\delta \leq .004$

Given $\lim_{x \to 2} (3x-1) = 5$ and $\varepsilon = .01$ find an appro. δ.

For $\varepsilon = .01$ we need to find a $\delta > 0 \ni$

when $|x-2| < \delta \longrightarrow |(3x-1)-5| < .01$

$$|3x-6| < .01$$

$$|x-2| < \frac{.01}{3}$$

$\therefore$ choose $\delta \leq \frac{.01}{3}$

Prove $\lim_{x \to 2} (3x-1) = 5$.

Pf: We need to show that $\forall \varepsilon > 0 \; \exists \delta > 0 \ni$

when $|x-2| < \delta \longrightarrow |(3x-1)-5| < \varepsilon$

$$|3x-6| < \varepsilon$$

$$|x-2| < \frac{\varepsilon}{3}$$

$\therefore$ Choose $\delta = \min\left\{1, \frac{\varepsilon}{3}\right\}$

Prove: $\lim_{x \to 4} (x^2 - 7x + 3) = -9$

Pf: We need to show that $\forall \varepsilon > 0 \ \exists \ \delta > 0 \ \ni$ when $|x-4| < \delta \rightarrow |(x^2 - 7x + 3) + 9| < \varepsilon$

$$|x^2 - 7x + 12| < \varepsilon$$

$$|x-4||x-3| < \varepsilon$$

$$|x-4| < \frac{\varepsilon}{|x-3|}$$

Consider $[3,5]$

For $x = 3 \rightarrow \frac{\varepsilon}{|x-3|} \ \nexists$

$x = 5 \rightarrow \frac{\varepsilon}{|x-3|} = \frac{\varepsilon}{2}$

$\therefore$ choose $\delta = \min\left\{1, \frac{\varepsilon}{2}\right\}$