Limits of Trigonometric Functions

But first...a quick review of domains and the greatest integer function

Find the domain of $f(x) = \dfrac{x-2}{x^2+5x+6}$.

$f \nexists$ when $x^2+5x+6=0 \longrightarrow x=-3$ or $x=-2$.

The domain of f is $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$.

Find the domain of $g(x)=\sqrt{x-4}$.

$g \exists$ when $x-4 \geq 0$

$\therefore$ the domain of g is $[4,\infty)$.

Find the domain of $h(x) = \sqrt{x^2+5x+6}$.

$h \; \exists$ when $x^2+5x+6 \geq 0$.

$x^2+5x+6 = 0 \longrightarrow x = -3$ or $x = -2$.

The domain of h is $(-\infty, -3] \cup [-2, \infty)$.

$f(x) = \|x\|$ "greatest integer" function

TI89

y1= floor(x)

$f(0) = 0$

$f(.5) = 0$

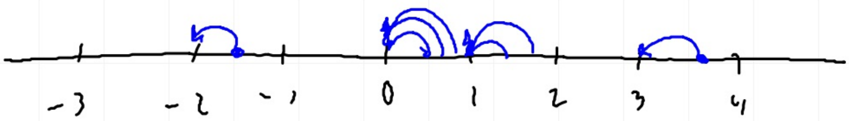
$f(.9) = 0$

$f(.999) = 0$

$f(1) = 1$

$f(6.3) = 6$

$f(-1.3) = -2$

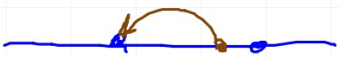


$f(1.1) = 1$

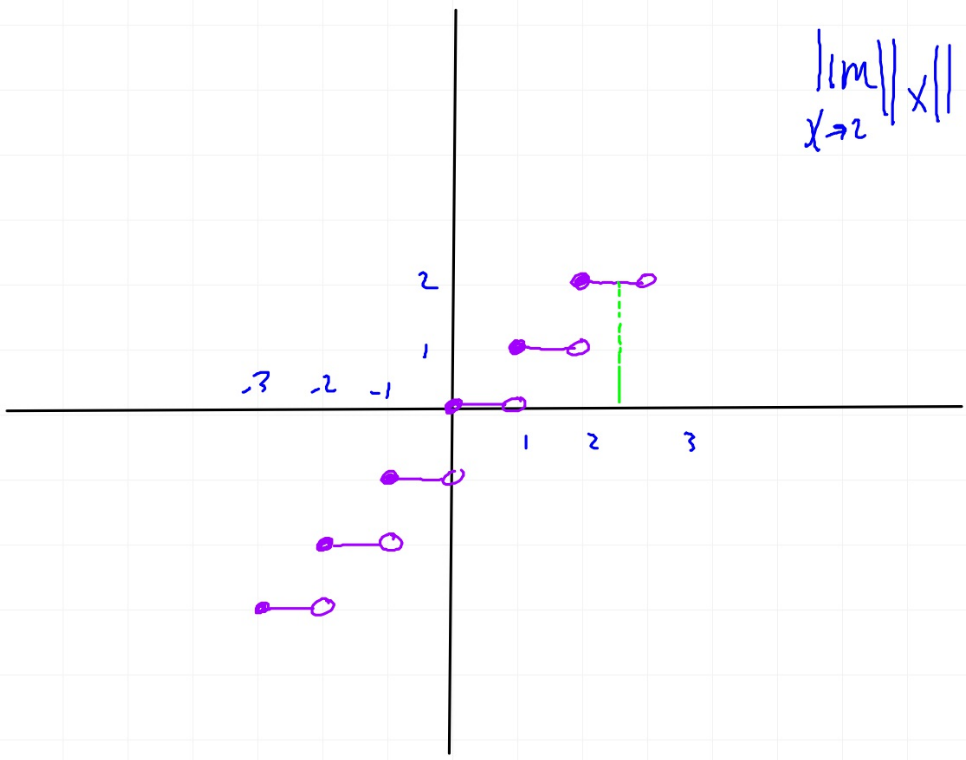
$f(1.8) = 1$

$f(2) = 2$

$f(2.9) = 2$



$\lim_{x \to 2} [\![x]\!] \quad \nexists$



$$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1} \qquad \boxed{\lim_{x \to 0} \frac{1-\cos x}{x} = 0}$$

$$\frac{\sin \star}{\star} \to 1 \qquad \frac{1-\cos \star}{\star} \to 0$$

$$\lim_{x \to 0} \frac{\sin 2x}{5x} = \lim_{x \to 0} \frac{\overbrace{\frac{\sin 2x}{2x}}^{1} \cdot 2\cancel{x}}{5\cancel{x}} = \frac{2}{5}$$

$$\lim_{x \to 0} \frac{7x}{\sin 3x} = \lim_{x \to 0} \frac{7x}{\underbrace{\frac{\sin 3x}{3x}}_{1} \cdot 3x} = \frac{7}{3}$$

$$\lim_{x\to 0} \frac{\sin 5x}{\sin 3x} = \lim_{x\to 0} \frac{\frac{\sin 5x}{5x}\cdot 5\cancel{x}}{\frac{\sin 3x}{3x}\cdot 3\cancel{x}} = \frac{5}{3}.$$

$$\lim_{x\to 0} \frac{x^2}{\sin^2 3x} = \lim_{x\to 0} \frac{\overset{1}{x^2}}{\frac{\sin 3x}{3x}\frac{\sin 3x}{3x}\cdot 9x^2} = \frac{1}{9}$$

$$\lim_{x\to 0} \frac{\tan x}{2x} = \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{2x} = \lim_{x\to 0} \frac{\sin x}{\cos x}\cdot\frac{1}{2x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x}\,\frac{1}{2\cos x} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{1-\cos 8x}{2x} = \lim_{x \to 0} \frac{\left(\frac{1-\cos 8x}{8x}\right)^{0} 8x}{2x} = 0$$