Limit Theorems

(How we really find limits!)

$$\lim_{x \to 2} (x^2+1) = 5$$

To find a limit as $x \to a$

(Plug in the "a")

Result of	Limit
C	C
$\frac{0}{0}$	not done → factor, rationalize, L'Hopital
$\frac{C}{0}$	$\nexists$ L & R $\pm\infty$

E1 $\lim_{x \to 5} (5x - 3)$

$$\lim_{x \to 5} (5x-3) = 22$$

E2 $\lim_{x \to 1} \frac{x^2 - x - 2}{x + 1}$

$$\lim_{x \to 1} \frac{x^2 - x - 2}{x+1} = -1$$

E3 $\lim_{x \to -3} \frac{x^2+5x+6}{x+3}$

$\lim_{x \to -3} \frac{\cancel{(x+3)}(x+2)}{\cancel{x+3}} = -1$

OR

$\lim_{x \to -3} \frac{x^2+5x+6}{x+3} = -1$

E4

$$\lim_{x \to 2} \frac{x^2 - 9}{x - 2}$$

$$\lim_{x \to 2} \frac{x^2 - 9}{x - 2} \nexists$$

$$\lim_{x \to 2^+} \frac{x^2 - 9}{x - 2} = -\infty$$

and $\lim_{x \to 2^-} \frac{x^2 - 9}{x - 2} = +\infty$

(E5) Find the VA (if any) of $f(x) = \frac{x^2-9}{x-2}$.

f has a VA at $x=2$ because

$f(2) \nexists$ and $\lim_{x \to 2} f(x) = \pm\infty$.

E6 $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3} = \lim_{x \to 9} \frac{\cancel{x - 9}}{\cancel{(x - 9)}(\sqrt{x} + 3)} = \frac{1}{6}$$

OR

$$\lim_{x \to 9} \frac{\cancel{\sqrt{x} - 3}}{\cancel{(\sqrt{x} - 3)}(\sqrt{x} + 3)} = \frac{1}{6}.$$

E7

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$$

$$\lim_{x \to 2} \frac{\frac{2-x}{2x}}{x-2} = \lim_{x \to 2} \frac{\overset{-1}{\cancel{2-x}}}{2x\cancel{(x-2)}} = -\frac{1}{4}$$

E8 Given $f(x) = \begin{cases} \sqrt{x-4} & x > 4 \\ 8-2x & x < 4 \end{cases}$ find $\lim_{x \to 4} f(x)$.

$\lim_{x \to 4} f(x) = 0$ because $\lim_{x \to 4^+} f(x) = 0$

and $\lim_{x \to 4^-} f(x) = 0$.

Ex 9 $\lim_{x \to 7} |x-7|$

$$|x-7| = \begin{cases} x-7 & x \geq 7 \\ 7-x & x < 7 \end{cases}$$

$\lim_{x \to 7} |x-7| = 0$ because $\lim_{x \to 7^+} |x-7| = 0$

and $\lim_{x \to 7^-} |x-7| = 0$.

Ex 11 Given $f(x) = \frac{|x-2|}{x-2}$ find $\lim_{x \to 2} f(x)$.

$$f(x) = \begin{cases} \frac{x-2}{x-2} & x > 2 \\ \frac{2-x}{x-2} & x < 2 \end{cases} = \begin{cases} 1 & x > 2 \\ -1 & x < 2 \end{cases}$$

$\lim_{x \to 2} f(x) \nexists$ because $\lim_{x \to 2^+} f(x) = 1$

but $\lim_{x \to 2^-} f(x) = -1$.