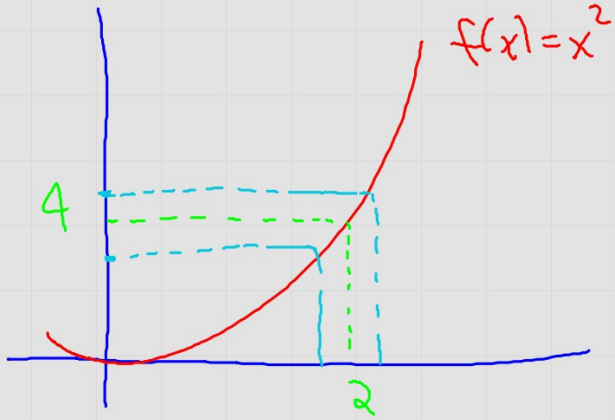
The Limit of a Function

- finding a limit from a graph
- estimating limits using tables
- vertical asymptotes

$\lim_{x \to a} f(x) = L$ means that we can generate function values as close to L as we want by using inputs sufficiently close to (but never equal to) $x = a$.



$x \to a$

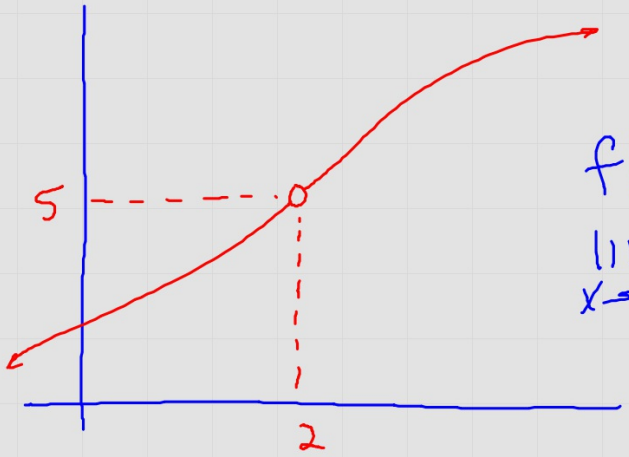
$x \to \infty$

$x \to -\infty$

$f(3) = 8$

$\lim_{x \to 3} f(x) = 8$

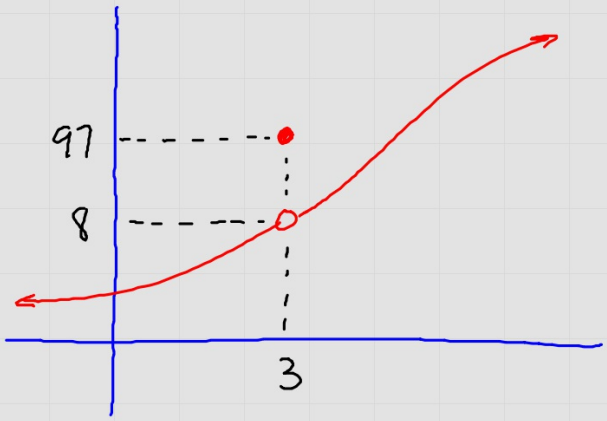
$\lim_{x \to 3} f(x) = 8$ because $\lim_{x \to 3^+} f(x) = 8$ and $\lim_{x \to 3^-} f(x) = 8$



$f(2) \nexists$

$\lim_{x \to 2} f(x) = 5$

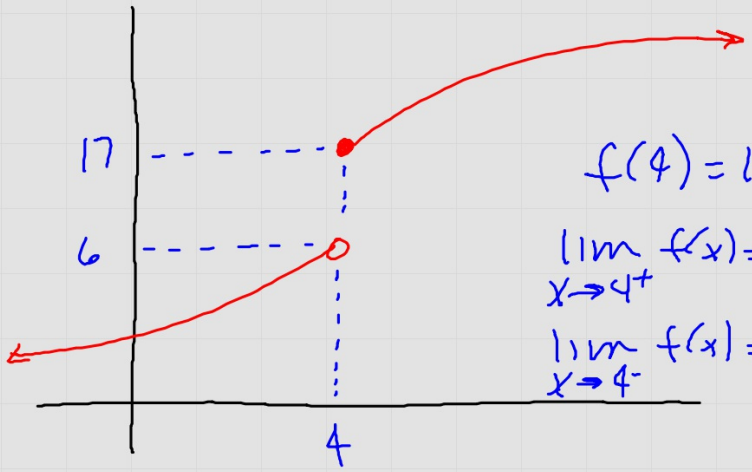
$\lim_{x \to 2} f(x) = 5$ because $\lim_{x \to 2^+} f(x) = 5$ and $\lim_{x \to 2^-} f(x) = 5$.



$f(3) = 97$

$\lim_{x \to 3} f(x) = 8$

$\lim_{x \to 3} f(x) = 8$ because $\lim_{x \to 3^+} f(x) = 8$ and $\lim_{x \to 3^-} f(x) = 8$.

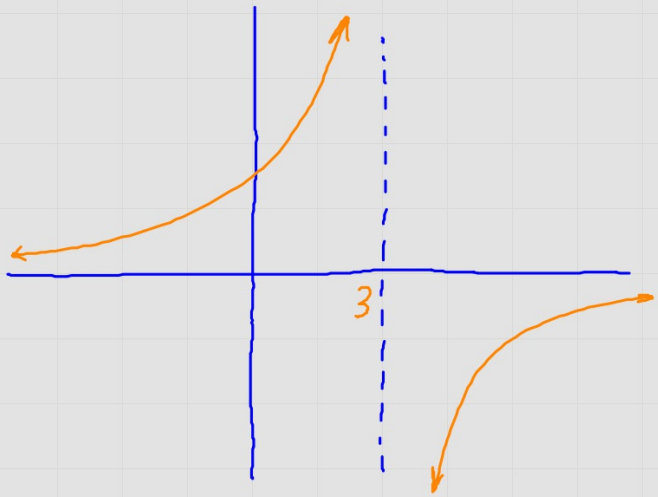


$f(4) = 17$

$\lim_{x \to 4^+} f(x) = 17$

$\lim_{x \to 4^-} f(x) = 6$

Since $\lim_{x \to 4^+} f(x) \neq \lim_{x \to 4^-} f(x) \longrightarrow \lim_{x \to 4} f(x) \nexists$.



$\lim_{x \to 3^+} f(x) = -\infty$ $\lim_{x \to 3^-} f(x) = +\infty$

$\lim_{x \to 3} f(x) \nexists$

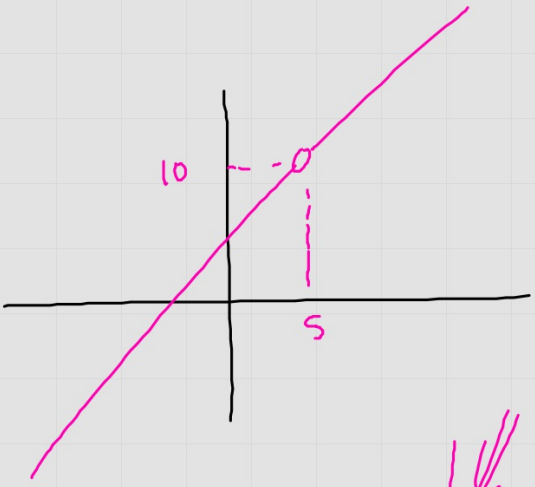
VA

f has a VA at $x=a$ iff

$f(a) \nexists$ and $\lim_{x \to a} f(x) = \pm\infty$.

$$f(x) = \frac{x^2 - 25}{x - 5}$$

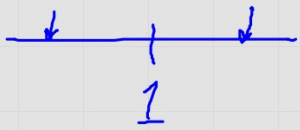
$$\frac{(x-5)(x+5)}{x-5}$$



$\frac{10}{4}$

"Howler"

Est. $\lim_{x \to 1} \frac{x-1}{x^2-1}$.



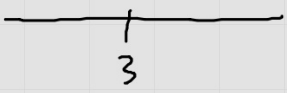
x	$\frac{x-1}{x^2-1}$
.500	.667
.900	.526
.990	.503
.999	.500

x	$\frac{x-1}{x^2-1}$
1.500	.400
1.100	.476
1.010	.498
1.001	.500

$\lim_{x \to 1} \frac{x-1}{x^2-1} = \frac{1}{2}$ because $\lim_{x \to 1^+} \frac{x-1}{x^2-1} = \frac{1}{2}$

and $\lim_{x \to 1^-} \frac{x-1}{x^2-1} = \frac{1}{2}$.

Est $\lim_{x \to 3} \frac{1}{x-3}$.



x	$\frac{1}{x-3}$
3.500	2
3.100	10
3.01	100
3.001	1000

x	$\frac{1}{x-3}$
2.500	-2
2.900	-10
2.990	-100
2.999	-1000

$\lim_{x \to 3} \frac{1}{x-3}$ DNE.

$\lim_{x \to 3^+} \frac{1}{x-3} = +\infty$ and $\lim_{x \to 3^-} \frac{1}{x-3} = -\infty$.

Est $\lim_{x \to 0} \sin \frac{\pi}{x}$