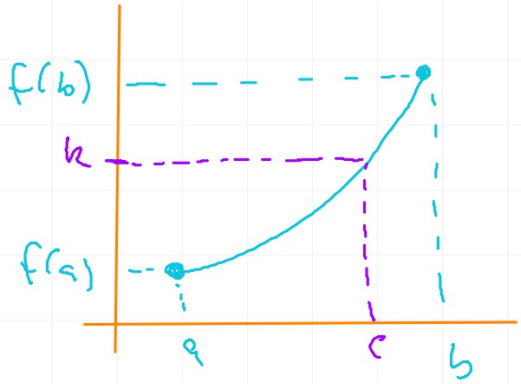
Intermediate Value Theorem

If f is cont on $[a,b]$ then

$\forall\ k \in (f(a), f(b))\ \exists\ c \in (a,b)$

$\ni\ f(c) = k$.



Given $f(x) = 2 + x - x^2$ on $[0,3]$ and $k = 1$,
find $c \ni f(c) = k$.

$$2 + x - x^2 = 1$$

$$1 + x - x^2 = 0 \longrightarrow x = -.618 \text{ or } x = 1.618.$$

$-.618 \notin (0,3) \therefore c = 1.618.$

Show that $f(x) = x^3 - 2x^2 + x - 5$ has a zero on $[2, 3]$.

Since $f(2) = -3$ and $f(3) = 7$
by IVT $f(x) = 0$ for some $x \in (2,3)$.

Show that $x^3 - 2x^2 + x - 5 = 0$ has a root between $x = 2$ and $x = 3$.

Let $f(x) = x^3 - 2x^2 + x - 5$

Since $f(2) = -3$ and $f(3) = 7$ by IVT $f(x) = 0$ for some $x \in (2,3)$ $\therefore$ $x^3 - 2x^2 + x - 5 = 0$ has a root between $x = 2$ and $x = 3$.