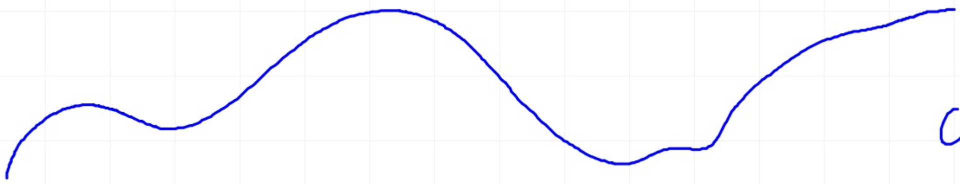
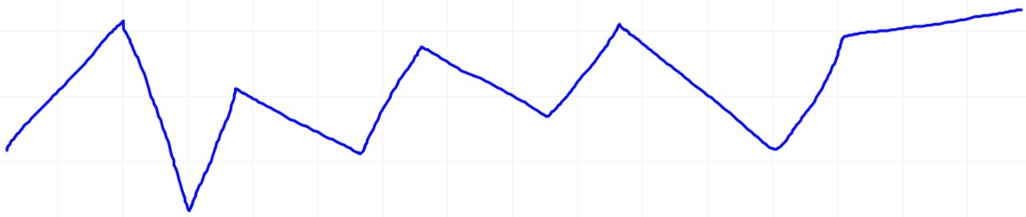
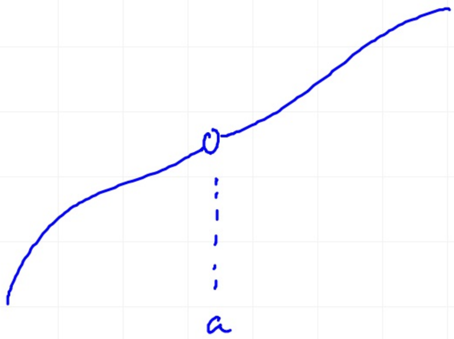
Continuity

cont. at a <u>number</u> / interval

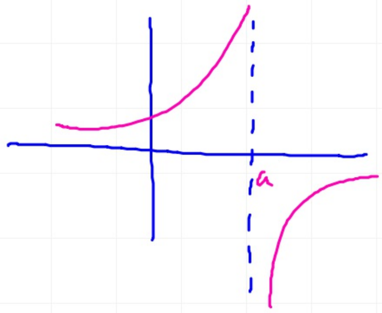


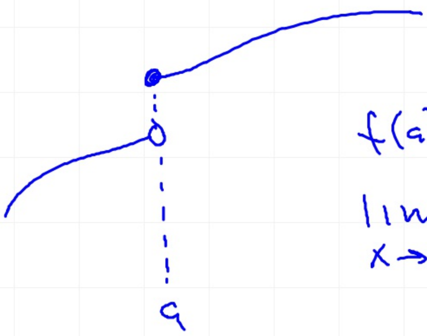
cont





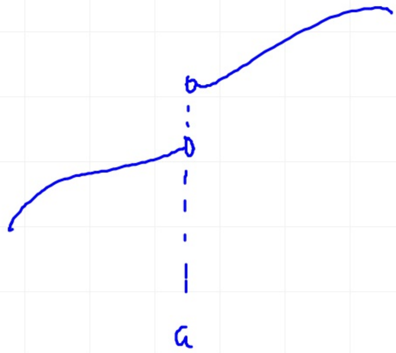
$f(a) \nexists$





$f(a)$ ✓

$\lim_{x \to a} f(x) \nexists$



f is cont. at $x=a$ iff

① $f(a)\ \exists$

② $\lim_{x\to a} f(x)\ \exists$

③ $f(a) = \lim_{x\to a} f(x)$

Removable if f has a limit as $x\to a$

Essential if $\lim_{x\to a} f(x)\ \nexists$

Is $f(x) = \dfrac{x^2+5x+2}{x-3}$ cont. at $x=3$?

Cont test at $x=3$

f is discont. at $x=3$ because $f(3)\ \nexists$.

Is $f(x) = \dfrac{x^2+2x-15}{x-3}$ cont at $x=3$? Determine if disc. is remov or essen. If removable, redefine f so that it is cont.

Cont. test at $x=3$

f is discont. at $x=3$ because $f(3)\ \nexists$.

Removable | Essen

The discon. is removable because $\lim_{x\to 3} f(x) = 8$.

Redefine f

$$f(x) = \begin{cases} \dfrac{x^2+2x-15}{x-3} & x \neq 3 \\ 8 & x = 3 \end{cases}$$

4
4

Is $f(x) = \begin{cases} x^2 & x < 2 \\ 2x & x \geq 2 \end{cases}$ cont at $x = 2$?

<u>Cont test at $x = 2$</u>

$f(2) = 4$

$\lim_{x \to 2} f(x) = 4$ because $\lim_{x \to 2^+} f(x) = 4$

and $\lim_{x \to 2^-} f(x) = 4$

$\therefore$ f is cont at $x = 2$ because $f(2) = \lim_{x \to 2} f(x)$.

Is $f(x) = \begin{cases} x^2 & x \le 3 \\ x-5 & x > 3 \end{cases}$ cont at $x=3$?

<u>Cont test at $x=3$</u>

$f(3) = 9$

$$\left.\begin{array}{l} \lim\limits_{x\to 3^+} f(x) = -2 \\ \lim\limits_{x\to 3^-} f(x) = 9 \end{array}\right\} \therefore \lim_{x\to 3} f(x) \nexists.$$

$\therefore$ f is discon. at $x=3$ because $\lim\limits_{x\to 3} f(x) \nexists$.

Determine where $f(x) = \begin{cases} x^2 - 49 & x \leq -7 \\ 49 - x^2 & -7 < x < 7 \\ 5 - x & x \geq 7 \end{cases}$ is disc.

Cont test at $x = -7$

$f(-7) = 0$

$$\left.\begin{aligned} \lim_{x \to -7^+} f(x) &= 0 \\ \lim_{x \to -7^-} f(x) &= 0 \end{aligned}\right\} \therefore \lim_{x \to -7} f(x) = 0$$

$\therefore f$ is cont at $x = -7$ becau

$f(-7) = \lim_{x \to -7} f(x)$.

Cont test at $x = 7$

$f(7) = -2$

$$\left.\begin{aligned} \lim_{x \to 7^+} f(x) &= -2 \\ \lim_{x \to 7^-} f(x) &= 0 \end{aligned}\right\} \therefore \lim_{x \to 7} f(x) \nexists$$

$\therefore f$ is disc. at $x = 7$

because $\lim_{x \to 7} f(x) \nexists$.

For what value of b will $f(x) = \begin{cases} \dfrac{x^2-25}{x-5} & x \neq 5 \\ b & x = 5 \end{cases}$ be cont at $x = 5$?

<u>Cont test at $x=5$</u>

$f(5) = b$

$\lim_{x \to 5} f(x) = 10$

f is cont at $x=5$ if $b=10$