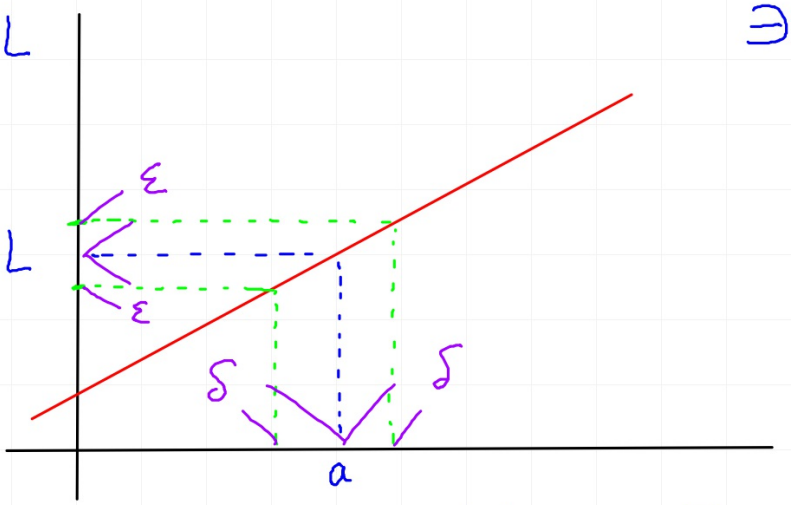
The Definition of Limit

$\lim_{x \to a} f(x) = L$

$\exists$ "such that"

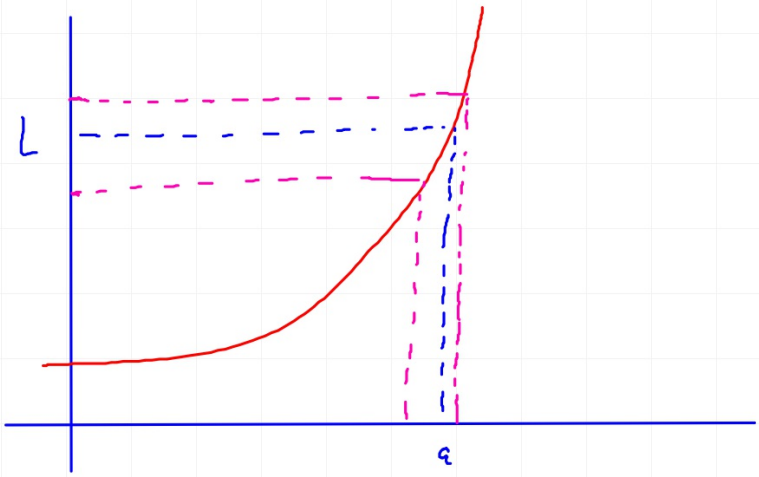


ε epsilon

δ delta

$\lim_{x \to a} f(x) = L$ is true if $\forall \varepsilon > 0 \; \exists \; \delta > 0 \; \ni$

whenever $|x - a| < \delta \longrightarrow |f(x) - L| < \varepsilon$.


L
q

For $\varepsilon = .01$ find an appropriate δ for $\lim_{x \to 3}(2x-1) = 5$.

$2x_1 - 1 = 5.01$ $\qquad$ $2x_2 - 1 = 4.99$

$2x_1 = 6.01$ $\qquad$ $2x_2 = 5.99$

$x_1 = 3.005$ $\qquad$ $x_2 = 2.995$

$\delta_1 = .005$ $\qquad$ $\delta_2 = .005$

$\therefore$ choose $\delta \leq .005$

Given $\lim_{x \to 3} (2x-1) = 5$ and $\varepsilon = .01$ find an appropriate δ.

For $\varepsilon = .01$ we need to find a $\delta > 0 \ni$ whenever $|x-3| < \delta \rightarrow |(2x-1)-5| < .01$

$$|2x-6| < .01$$

$$|x-3| < \frac{.01}{2}$$

$\therefore$ Choose $\delta \leq \frac{.01}{2}$

Given $\lim_{x \to 2} (3x+4) = 10$ and $\varepsilon = .01$ find an appropriate δ.

For $\varepsilon = .01$ we need to find a $\delta > 0 \ni$ when $|x-2| < \delta \longrightarrow |(3x+4)-10| < .01$

$$|3x-6| < .01$$

$$|x-2| < \frac{.01}{3}$$

$\therefore$ Choose $\delta \leq \frac{.01}{3}$.

Prove $\lim_{x \to 2} (3x+4) = 10$.

Pf: We need to show that $\forall \varepsilon > 0 \; \exists \delta > 0 \ni$
when $|x-2| < \delta \rightarrow |(3x+4)-10| < \varepsilon$

$$|3x-6| < \varepsilon$$

$$|x-2| < \frac{\varepsilon}{3}$$

$\therefore$ choose $\delta = \min\left\{1, \frac{\varepsilon}{3}\right\}$

Prove $\lim_{x \to 2} (x^2 + 7x - 3) = 15$.

Pf: We need to show that $\forall \varepsilon > 0 \; \exists \delta > 0 \ni$

when $|x - 2| < \delta \longrightarrow |(x^2 + 7x - 3) - 15| < \varepsilon$

$|x^2 + 7x - 18| < \varepsilon$

$|x - 2||x + 9| < \varepsilon$

$|x - 2| < \frac{\varepsilon}{|x+9|}$

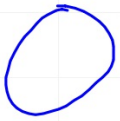
Consider $[1, 3]$

If $x = 1 \rightarrow \frac{\varepsilon}{|x+9|} = \frac{\varepsilon}{10}$

$x = 3 \rightarrow \frac{\varepsilon}{|x+9|} = \frac{\varepsilon}{12}$

$\therefore$ Choose $\delta = \min\{1, \frac{\varepsilon}{12}\}$

Find a δ

$\therefore$ choose $\delta \leq$ 

Prove

$\therefore$ choose $\delta = \min\{1, \varepsilon\}$.