Limits of Trigonometric Functions

But first...a quick review of domains and the greatest integer function

Find the domain of $f(x) = \dfrac{x-2}{x^2+5x+6}$.

$f \nexists$ when $x^2+5x+6=0 \longrightarrow x=-3$ or $x=-2$.

$\therefore$ the domain of f is $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$.

Find the domain of $f(x) = \sqrt{x-5}$.

$f \ \exists$ when $x-5 \geq 0$.

$\therefore$ the domain of f is $[5, \infty)$

Find the doman of $g(x) = \sqrt{x^2-4}$

$g \ \exists$ when $x^2-4 \geq 0$

$x^2-4=0 \rightarrow x=-2$ or $x=2$

$\therefore$ the domain of g is $(-\infty,-2] \cup [2,\infty)$.

$f(x) = ||x||$

"greatest integer function"

$f(0) = 0$

$f\left(\frac{1}{2}\right) = 0$

$f(.9) = 0$

$f(1) = 1$

$f(1.5) = 1$

$f(1.999) = 1$

$f(2) = 2$

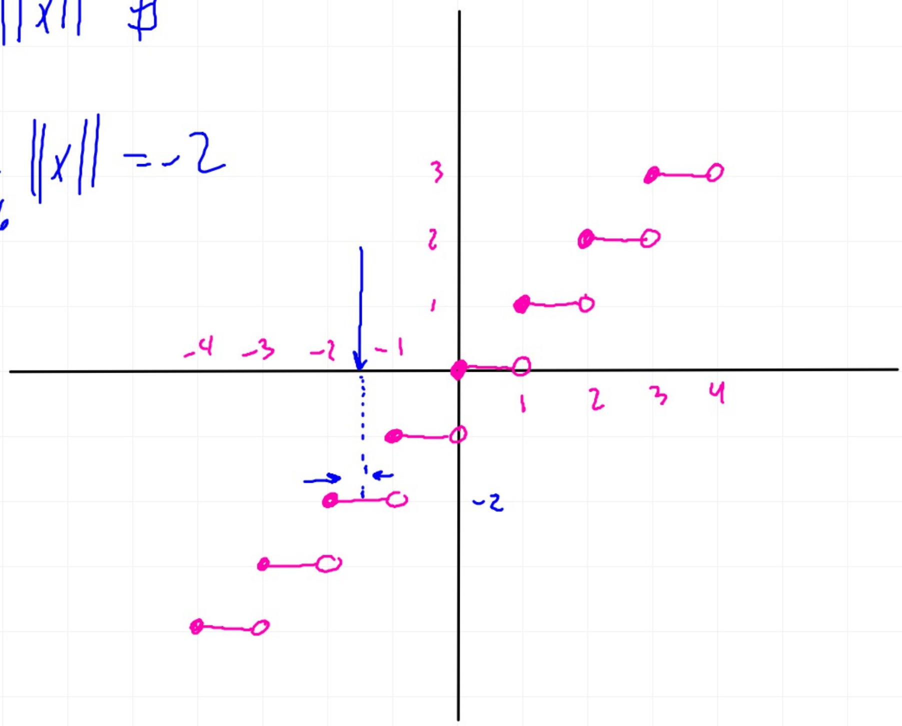
$f(-1.3) = -2$

TI-89

floor(x)

$$\lim_{x \to 2} \|x\| \ \nexists$$

$$\lim_{x \to -1.6} \|x\| = -2$$



$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{1-\cos x}{x} = 0$$

$$\frac{\sin \star}{\star} \to 1 \qquad \frac{1-\cos \star}{\star} \to 0$$

$$\lim_{x \to 0} \frac{\sin 2x}{2x} = 1$$

$$\lim_{x \to 0} \frac{\sin 2x}{5x} = \lim_{x \to 0} \frac{\overset{1}{\frac{\sin 2x}{2x}} \cdot 2\cancel{x}}{5\cancel{x}} = \frac{2}{5}$$

$$\lim_{x \to 0} \frac{\sin 8x}{2x} = \lim_{x \to 0} \frac{\frac{\sin 8x}{8x} \cdot 8\cancel{x}}{2x} = 4$$

$$\lim_{x \to 0} \frac{9x}{\sin 7x} = \lim_{x \to 0} \frac{9x}{\frac{\sin 7x}{7x} \cdot 7x} = \frac{9}{7}$$

$$\lim_{x \to 0} \frac{\sin 3x}{\sin 2x} = \lim_{x \to 0} \frac{\frac{\sin 3x}{3x} \cdot 3x}{\frac{\sin 2x}{2x} \cdot 2x} = \frac{3}{2}$$

$$\lim_{x \to 0} \frac{\sin 3x}{\sin 2x} = \frac{3}{2}$$

$$\lim_{x\to 0} \frac{x^2}{\sin^2 3x} = \lim_{x\to 0} \frac{\cancel{x^2}}{\frac{\sin 3x}{3x}\frac{\sin 3x}{3x} \cdot 9\cancel{x^2}} = \frac{1}{9}$$

$$\lim_{x\to 0} \frac{\tan x}{2x} = \lim_{x\to 0} \frac{\sin x}{\cos x} \frac{1}{2x}$$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{x} x}{\cos x} \cdot \frac{1}{2x}$$

$$= \frac{1}{2}$$

$$\lim_{x \to 0} \frac{1-\cos 3x}{7x} = \lim_{x \to 0} \frac{\overset{0}{\frac{1-\cos 3x}{3x}} \cdot 3\cancel{x}}{7\cancel{x}} = 0$$