Limit Theorems

(How we really find limits!)

$$\lim_{x \to 2} (x^2 + 3) = 7 \qquad \lim_{x \to 3} \frac{x^2 - 3}{x - 3}$$

How to find a limit

Plug in the "a"

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

Result		Limit
c	$\longrightarrow$	c
$\frac{0}{0}$	$\longrightarrow$	not done: factor, rationalize, L'Hopital's Rule
$\frac{c}{0}$	$\longrightarrow$	$\nexists < \$R \neq \infty$

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}=\lim_{x\to 2}(x+2)=4$$

$$\left[\ \frac{x^2-4}{x-2}=\frac{x+2\quad x-2}{x-2}\right.$$

$$\lim_{x\to 2}\frac{x+3}{x-2}\ \nexists$$

$$\lim_{x\to 2^+}\frac{x+3^{+}}{x-2_{+}}=+\infty \text{ and } \lim_{x\to 2^-}\frac{x+\overset{+}{3}}{\underset{-}{x-2}}=-\infty$$

1. $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \, \frac{\sqrt{x} + 3}{\sqrt{x} + 3} = \lim_{x \to 9} \frac{x - 9}{(x-9)(\sqrt{x} + 3)} = \frac{1}{6}$$

OR

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{(\sqrt{x} - 3)(\sqrt{x} + 3)} = \frac{1}{6}.$$

OR

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = \frac{1}{6}$$

2. $\lim_{x \to 5} (x^2+1)$

$$\lim_{x \to 5} (x^2+1) = 26.$$

3. $\lim_{x \to -3} \frac{x^2-x-12}{x+3}$

$$\lim_{x \to -3} \frac{(x+3)(x-4)}{x+3} = -7$$

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$$

$$\lim_{x \to 2} \frac{\frac{2 - x}{2x}}{x - 2} = \lim_{x \to 2} \frac{\overset{-1}{\cancel{2 - x}}}{2x\,(\underset{1}{\cancel{x - 2}})} = -\frac{1}{4}$$

Find the VA (if any) of $f(x) = \dfrac{x+6}{x-5}$.

f has a VA at $x = 5$ because

$f(5) \nexists$ and $\lim_{x \to 5} f(x) = \pm\infty$.

Find the VA (if any) of $g(x) = \dfrac{x^2-9}{x-3}$.

g has no VA at $x = 3$ because

$\lim_{x \to 3} g(x) = 6$.

Given $f(x) = \begin{cases} \sqrt{x} - 4 & x > 4 \\ 8 - 2x & x < 4 \end{cases}$ find $\lim_{x \to 4} f(x)$.

$\lim_{x \to 4} f(x) \nexists$ because $\lim_{x \to 4^+} f(x) = -2$

but $\lim_{x \to 4^-} f(x) = 0$.

Given $g(x) = |x-7|$ find $\lim\limits_{x \to 7} g(x)$.

$$g(x) = \begin{cases} x-7 & x \geq 7 \\ 7-x & x < 7 \end{cases}$$

$\lim\limits_{x \to 7} g(x) = 0$ because $\lim\limits_{x \to 7^+} g(x) = 0$

and $\lim\limits_{x \to 7^-} g(x) = 0$.

Given $f(x) = \dfrac{|x-2|}{x-2}$ find $\lim\limits_{x \to 2} f(x)$.

$$f(x) = \begin{cases} \dfrac{x-2}{x-2} & x > 2 \\ \dfrac{2-x}{x-2} & x < 2 \end{cases} = \begin{cases} 1 & x > 2 \\ -1 & x < 2 \end{cases}.$$

$\lim\limits_{x \to 2} f(x) \nexists$ because $\lim\limits_{x \to 2^+} f(x) = 1$

but $\lim\limits_{x \to 2^-} f(x) = -1$.