The Limit of a Function

- finding a limit from a graph
- estimating limits using tables
- vertical asymptotes

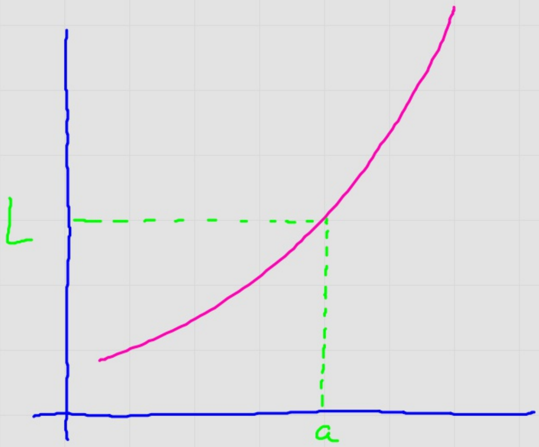
$x \to a$

$x \to \infty$

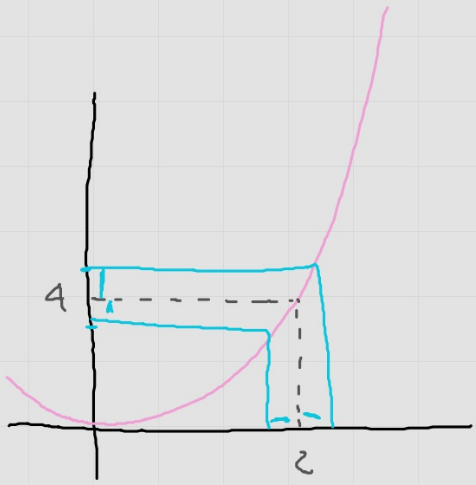
$$\lim_{x \to 3} (x^2 - 2)$$

$$\lim_{x \to \infty} \frac{1}{x-1}$$

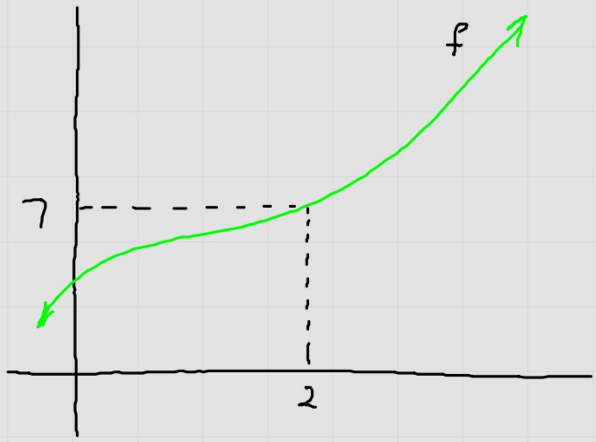
$$\lim_{x \to -\infty} \frac{3}{x-5}$$



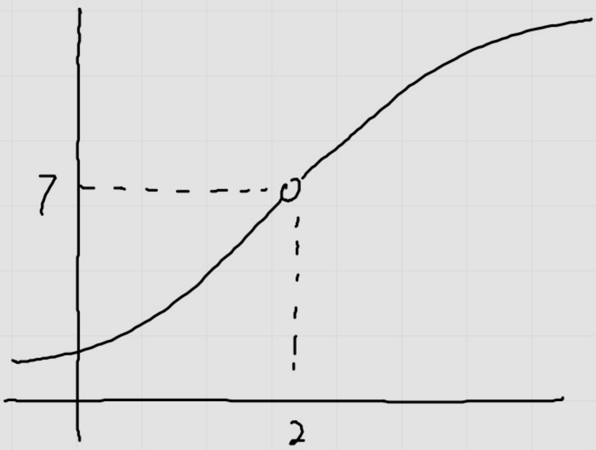
$$\lim_{x \to a} f(x) = L$$



$$\lim_{x \to 2} x^2 = 4$$

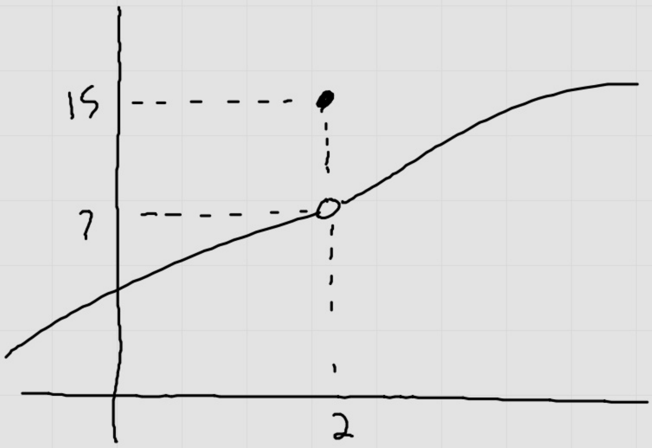


$$\lim_{x \to 2} f(x) = 7 \qquad f(2) = 7$$

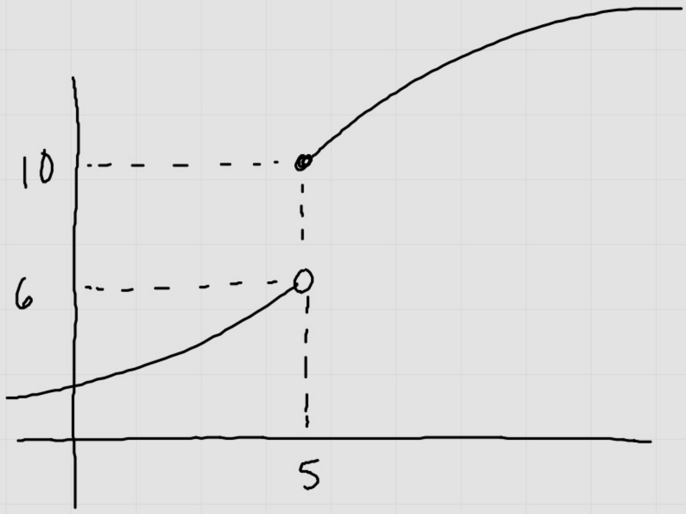


A function does not have to exist at $x=a$ to have a limit as $x \to a$

$$\lim_{x \to 2} f(x) = 7$$

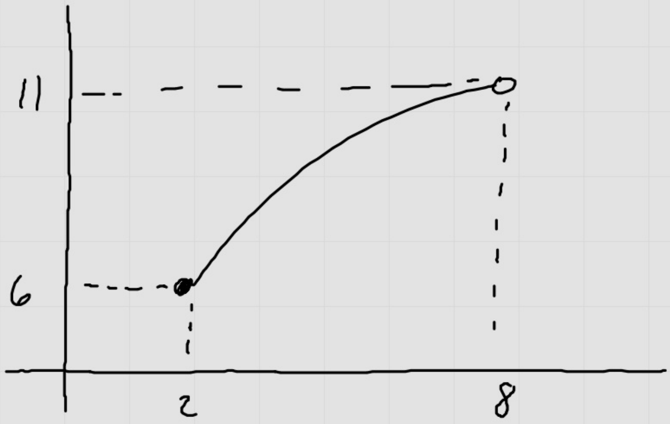


$f(2) = 15$ $\qquad$ $\lim_{x \to 2} f(x) = 7$



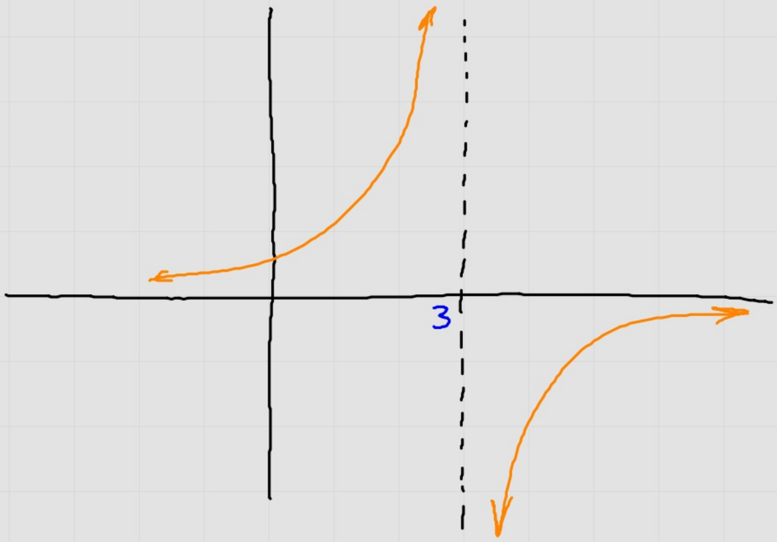
$$\lim_{x \to 5^+} f(x) = 10 \quad \text{but} \quad \lim_{x \to 5^-} f(x) = 6$$

$$\therefore \lim_{x \to 5} f(x) \nexists$$

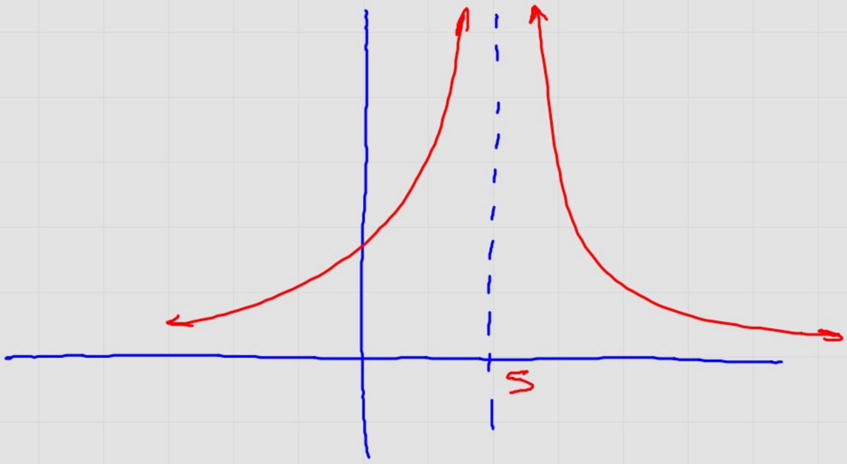


$$\lim_{x \to 8^-} f(x) = 11 \qquad \lim_{x \to 8^+} f(x) \nexists$$

$$\therefore \lim_{x \to 8} f(x) \nexists$$



$$\lim_{x \to 3^-} f(x) = \infty \qquad \lim_{x \to 3^+} f(x) = -\infty$$



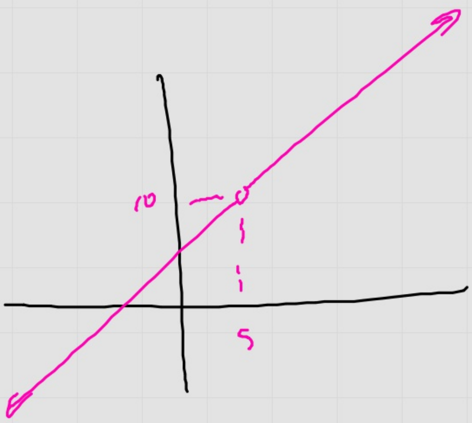
$$\lim_{x \to 5^+} f(x) = +\infty \qquad \lim_{x \to 5^-} f(x) = +\infty$$

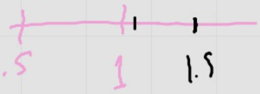
VA

f has a VA at $x=a$ iff
$f(a)\ \nexists$ and $\lim_{x\to a} f(x) = \pm\infty$

$$f(x) = \frac{x^2-25}{x-5}$$

$$\frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}}$$





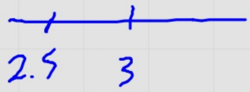
Estimate $\lim_{x \to 1} \frac{x-1}{x^2-1}$.

x	$\frac{x-1}{x^2-1}$
.500	.667
.900	.526
.990	.503
.999	.500

x	$\frac{x-1}{x^2-1}$
1.500	.400
1.100	.476
1.010	.498
1.001	.500

Since $\lim_{x \to 1^-} \frac{x-1}{x^2-1} = \frac{1}{2}$ and $\lim_{x \to 1^+} \frac{x-1}{x^2-1} = \frac{1}{2}$ $\rightarrow$ $\lim_{x \to 1} \frac{x-1}{x^2-1} = \frac{1}{2}$.

Est. $\lim_{x \to 3} \frac{1}{x-3}$



x	$\frac{1}{x-3}$
2.500	-2
2.900	-10
2.990	-100
2.999	-1000

$$\lim_{x \to 3^-} \frac{1}{x-3} = -\infty$$

x	$\frac{1}{x-3}$
3.500	2
3.100	10
3.010	100
3.001	1000

$$\lim_{x \to 3^+} \frac{1}{x-3} = +\infty$$

Est $\lim_{x \to 0} \sin \frac{\pi}{x}$