t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

   (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6} = 1.017$$

The temperature of the water is increasing at 1.017 °F/min at $t = 12$ min.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

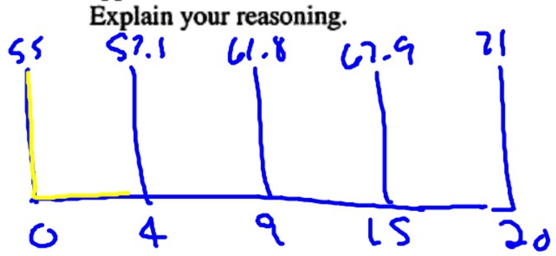
(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71 - 55 = 16$$

The temperature of the water increased 16°F from t=0min to t=20min.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.



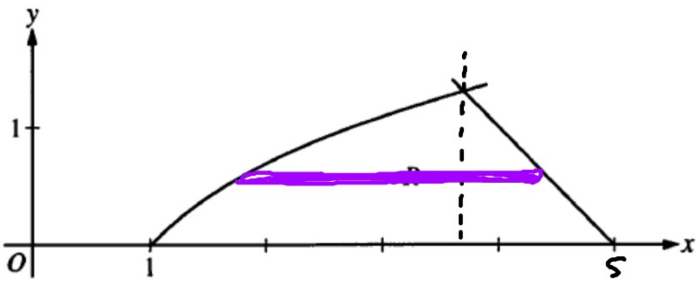
$$\frac{1}{20}\int_0^{20} w(t)\,dt = \frac{1}{20}\left[55(4) + 57.1(5) + 61.8(6) + 67.9(5)\right]$$

This approximation underestimates the avg temp. because w(t) is strictly increasing.

(d) For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$$71 + \int_{20}^{25} W'(t)\,dt = 73.043$$

$$\therefore 73.043^{\circ}F.$$



2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

<u>Isect</u>

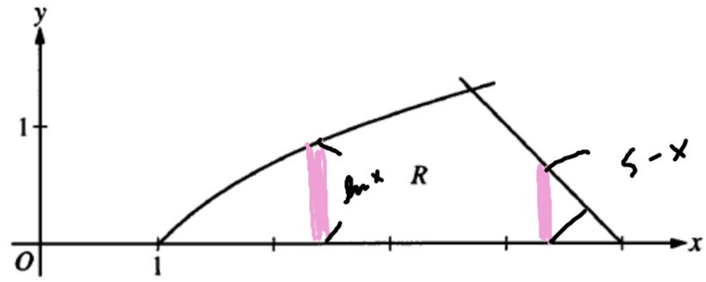
$\ln x = 5 - x$

$x = 3.693$

$y = 1.307$

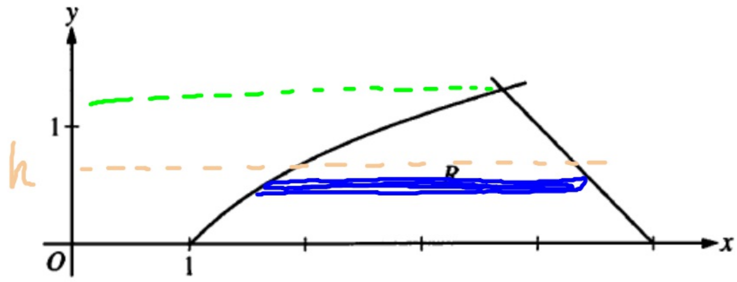
$$A = \int_{1}^{3.693} \ln x \, dx + \int_{3.693}^{5} (5-x)\,dx = 2.986$$

$$A = \int_{0}^{1.307} \left[(5-y) - (e^{y}) \right] dy$$



(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

$$V = \int_{1}^{3.693} [\ln x]^2 \, dx + \int_{3.693}^{5} (5-x)^2 \, dx$$



$x = 5 - y$

$x = e^y$

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

1 sect

$e^y = 5 - y$

$y = 1.307$

$$\int_0^k (5 - y - e^y)\,dy = \int_k^{1.307} (5 - y - e^y)\,dy$$

$$\int_0^k (5 - y - e^y)\,dy = \frac{2.986}{2}$$

13. Let f be a differentiable function such that $f(0) = -5$ and $f'(x) \leq 3$ for all x. Of the following, which is not a possible value for $f(2)$?

(A) -10 (B) -5 (C) 0 (D) 1 (E) 2

$$\frac{f(2) - f(0)}{2} \leq 3$$

$$f(2) + 5 \leq 6$$

$$f(2) \leq 1$$

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

$$f(x) = \begin{cases} x + b & \text{if } x \le 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

Cont test & diff test

14. Let f be the function given above. What are all values of a and b for which f is differentiable at $x = 1$?

(A) $a = \frac{1}{2}$ and $b = -\frac{1}{2}$

(B) $a = \frac{1}{2}$ and $b = \frac{3}{2}$

(C) $a = \frac{1}{2}$ and b is any real number

(D) $a = b + 1$, where b is any real number

(E) There are no such values of a and b.

$$f'(x) = \begin{cases} 1 & x < 1 \\ 2ax & x > 1 \end{cases}$$

$f'_+(1) = 2a \quad f'_-(1) = 1$

$2a = 1$

$a = \frac{1}{2}$

$f(1) = 1 + b$

$\lim_{x \to 1^+} f(x) = a$

$\lim_{x \to 1^-} f(x) = 1 + b$

$1 + b = a$

$1 + b = \frac{1}{2}$

$b = -\frac{1}{2}$

$f(3)$	$g(3)$	$f'(3)$	$g'(3)$
-1	2	5	-2

15. The table above gives values for the functions f and g and their derivatives at $x = 3$. Let k be the function given by $k(x) = \dfrac{f(x)}{g(x)}$, where $g(x) \neq 0$. What is the value of $k'(3)$?

(A) $-\frac{5}{2}$ (B) -2 (C) 2 (D) 3 (E) 8

$$k'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$k'(3) = \frac{(2)(5) - (-1)(-2)}{4}$$

$$= \frac{10-2}{4}$$

16. If $y = 5x\sqrt{x^2+1}$, then $\frac{dy}{dx}$ at $x = 3$ is

(A) $\frac{5}{2\sqrt{10}}$ (B) $\frac{15}{\sqrt{10}}$ (C) $\frac{15}{2\sqrt{10}} + 5\sqrt{10}$ (D) $\frac{45}{\sqrt{10}} + 5\sqrt{10}$ (E) $\frac{45}{\sqrt{10}} + 15\sqrt{10}$

$$\frac{dy}{dx} = (5x)\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} + 5\sqrt{x^2+1}$$

$$\left.\frac{dy}{dx}\right|_{x=3} = (15)\frac{3}{\sqrt{10}} + 5\sqrt{10}$$

$$= \frac{45}{\sqrt{10}} + 5\sqrt{10}$$

17. If $\lim_{h\to 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a ?

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{1}{2}$ (E) 2

$f(x) = \sin^{-1} x \longrightarrow \frac{1}{\sqrt{1-x^2}}$

$f'(a) = \longrightarrow \frac{1}{\sqrt{1-a^2}} = 2$

$2\sqrt{1-a^2} = 1$

$\sqrt{1-a^2} = \frac{1}{2}$

$1 - a^2 = \frac{1}{4}$

$a^2 = \frac{3}{4}$

$a = \pm \frac{\sqrt{3}}{2}$

$\sin^{-1} A \rightarrow \frac{A'}{\sqrt{1-A^2}}$

18. If $\ln(2x+y) = x+1$, then $\frac{dy}{dx} =$

(A) -2 (B) $2x+y-2$ (C) $2x+y$ (D) $4x+2y-2$ (E) $y-\frac{y}{x}$

$$\frac{2+\frac{dy}{dx}}{2x+y} = 1$$

$$2+\frac{dy}{dx} = 2x+y$$

$$\frac{dy}{dx} = 2x+y-2$$

$$\ln \star \longrightarrow \frac{\star'}{\star}$$