23. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{2xy}{x^2+1}$ whose graph contains the point $(0, 1)$?

(A) $y = e^{x^2}$

(B) $y = x^2 + 1$

(C) $y = \ln(x^2 + 1)$

(D) $y = 1 + \ln(x^2 + 1)$

(E) $y = \sqrt{1 + 2\ln(x^2 + 1)}$

$$\frac{1}{y}\,dy = \frac{2x}{x^2+1}\,dx$$

$$\ln|y| = \ln|x^2+1| + C.$$

$$\ln 1 = \ln 1 + C \rightarrow C = 0$$

$$\ln|y| = \ln|x^2+1|$$

$$y = x^2+1$$

$u = x^2+1$

$du = 2x\,dx$

$\int \frac{1}{u}\,du$

24. Sand is deposited into a pile with a circular base. The volume V of the pile is given by $V = \frac{r^3}{3}$, where r is the radius of the base, in feet. The circumference of the base is increasing at a constant rate of 5π feet per hour. When the circumference of the base is 8π feet, what is the rate of change of the volume of the pile, in cubic feet per hour?

(A) $\frac{8}{\pi}$ (B) 16 (C) 40 (D) 40π (E) 80π

$\frac{dC}{dt} = 5\pi$

$V = \frac{1}{3}r^3$

$\frac{dV}{dt} = r^2 \frac{dr}{dt}$

$= (16)\frac{5}{2}$

$= 40$

$C = 2\pi r$

$8\pi = 2\pi r$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$5\pi = 2\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{5}{2}$

25. $\lim_{h \to 0} \frac{e^{-1-h} - e^{-1}}{h}$ is

(A) -1 (B) $\frac{-1}{e}$ (C) 0 (D) $\frac{1}{e}$ (E) nonexistent

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$e^{☆} \to ☆' e^{☆}$

$\frac{-e^{-1-h}}{1} \to -e^{-1} \to -\frac{1}{e}$

L. Hop

26. Let f be the function given by $f(x) = x^3 + 5x$. For what value of x in the closed interval $[1,3]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) $\sqrt{\frac{7}{3}}$ (B) $\sqrt{\frac{13}{3}}$ (C) $\sqrt{5}$ (D) $\sqrt{6}$ (E) $\sqrt{\frac{19}{3}}$

$$3x^2 + 5 = \frac{f(3) - f(1)}{3-1}$$

$$3x^2 + 5 = \frac{42 - 6}{2}$$

$$3x^2 + 5 = 18$$

$$3x^2 = 13$$

$$x = \pm\sqrt{\frac{13}{3}}$$

27
13
42

27. If $e^{xy} - y^2 = e - 4$, then at $x = \frac{1}{2}$ and $y = 2$, $\frac{dy}{dx} =$

(A) $\frac{e}{4}$ (B) $\frac{e}{2}$ (C) $\frac{4e}{8-e}$ (D) $\frac{4e}{4-e}$ (E) $\frac{8-4e}{e}$

$$e^{xy}\left[x\frac{dy}{dx} + y\right] - 2y\frac{dy}{dx} = 0$$

$$e\left[\frac{1}{2}\frac{dy}{dx} + 2\right] - 2(2)\frac{dy}{dx} = 0$$

$$\frac{e}{2}\frac{dy}{dx} + 2e - 4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2e}{\frac{e}{2} - 4} = \frac{-4e}{e-8}\,\frac{-1}{-1} = \frac{4e}{8-e}$$

28. Let f be the function defined by $f(x) = x^3 + x^2 + x$. Let $g(x) = f^{-1}(x)$, where $g(3) = 1$. What is the value of $g'(3)$?

(A) $\frac{1}{39}$ (B) $\frac{1}{34}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) 39

$\frac{1}{f'(1)}$

$3x^2 + 2x + 1$

$\frac{1}{6}$

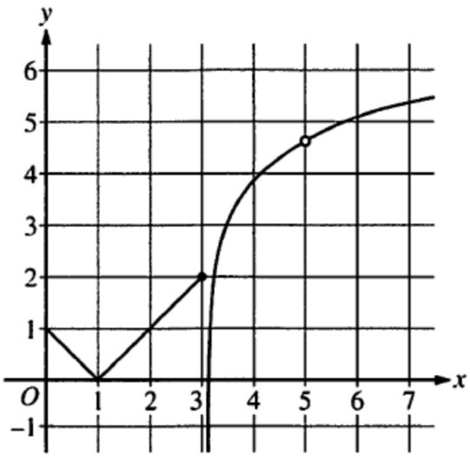
If (c,d) on $f \rightarrow (f^{-1})'(d) = \frac{1}{f'(c)}$

$(f^{-1})'(3) = \frac{1}{f'(1)}$

$g(3) = 1$ $(1,3)$ g

$(3,1)$ g^{-1}

$(3,1)$ f



Graph of f

76. The graph of a function f is shown above. Which of the following limits does not exist?

(A) $\lim_{x \to 1^-} f(x)$ 0

(B) $\lim_{x \to 1} f(x)$ 0

(C) $\lim_{x \to 3^-} f(x)$ 2

(D) $\lim_{x \to 3} f(x)$

(E) $\lim_{x \to 5} f(x)$

77. Let f be a function that is continuous on the closed interval $[1, 3]$ with $f(1) = 10$ and $f(3) = 18$. Which of the following statements must be true?

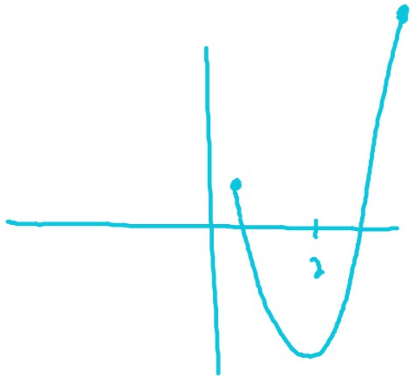
(A) $10 \le f(2) \le 18$

(B) f is increasing on the interval $[1, 3]$.

(C) $f(x) = 17$ has at least one solution in the interval $[1, 3]$.

(D) $f'(x) = 8$ has at least one solution in the interval $(1, 3)$.

(E) $\int_1^3 f(x)\,dx > 20$



78. Let R be the region bounded by the graphs of $y = e^x$, $y = e^3$, and $x = 0$. Which of the following gives the volume of the solid formed by revolving R about the line $y = -1$?

(A) $\pi\int_0^3 \left(e^3 - e^x + 1\right)^2 dx$

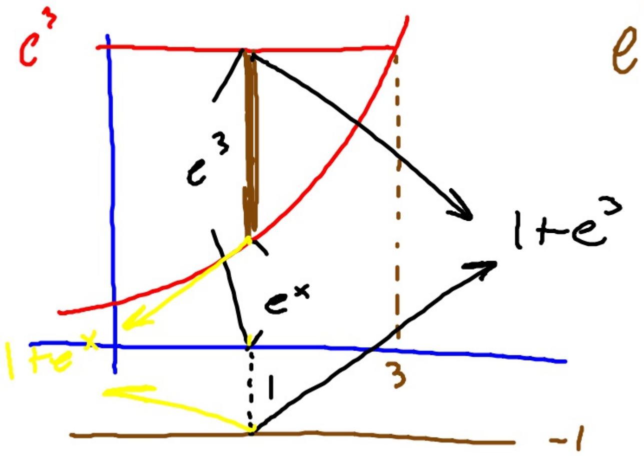
(B) $\pi\int_0^3 \left(e^3 - e^x - 1\right)^2 dx$

(C) $\pi\int_0^3 \left[\left(e^3 - e^x\right)^2 + 1\right] dx$

(D) $\pi\int_0^3 \left[\left(e^3 - e^x\right)^2 - 1\right] dx$

(E) $\pi\int_0^3 \left[\left(e^3 + 1\right)^2 - \left(e^x + 1\right)^2\right] dx$

$e^3 = e^x$



$$\pi\int_0^3 \left[(1+e^3)^2 - (1+e^x)^2\right] dx$$

79. The number of people who have entered a museum on a certain day is modeled by a function $f(t)$, where t is measured in hours since the museum opened that day. The number of people who have left the museum since it opened that same day is modeled by a function $g(t)$. If $f'(t) = 380(1.02^t)$ and $g'(t) = 240 + 240\sin\left(\frac{\pi(t-4)}{12}\right)$, at what time t, for $1 \le t \le 11$, is the number of people in the museum at a maximum?

(A) 1 (B) 7.888 (C) 9.446 (D) 10.974 (E) 11

$f(t) - g(t)$

$f'(t) - g'(t) = 0$

y3 = y1(x) - y(2) xmin 1

xmax 11

find zero where above to below

x	0	1	2	3
$f(x)$	5	2	3	6
$f'(x)$	−3	1	3	4

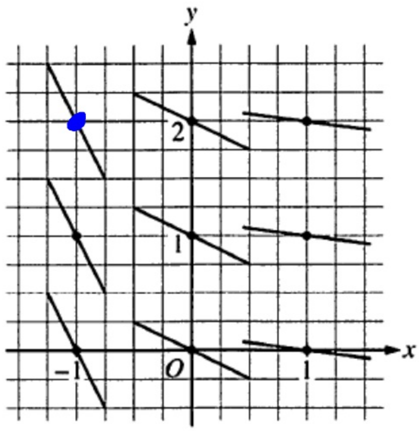
80. The derivative of the function f is continuous on the closed interval $[0, 4]$. Values of f and f' for selected values of x are given in the table above. If $\int_0^4 f'(t)\,dt = 8$, then $f(4) =$

(A) 0 (B) 3 (C) 5 (D) 10 (E) 13

$$\int_0^4 f'(x)\,dx = f(4) - f(0)$$

$$8 = f(4) - 5$$

$$f(4) = 13$$



81. A slope field for a differential equation is shown in the figure above. If $y = f(x)$ is the particular solution to the differential equation through the point $(-1, 2)$ and $h(x) = 3x \cdot f(x)$, then $h'(-1) =$

(A) −6 (B) −2 (C) 0 (D) 1 (E) 12

$h'(x) = 3x\,f'(x) + 3f(x)$

$h'(-1) = -3f'(-1) + 3f(-1)$

$= (-3)(-2) + (3)(2)$

$= 12$