76. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = t^2 \ln(t + 2)$. What is the acceleration of the particle at time $t = 6$?

(A) 1.500 (B) 20.453 (C) 29.453 (D) 74.860 (E) 133.417

y1 = v(t)

y2 = d(y1(x), x)

y2(6).

77. If $\int_0^3 f(x)\,dx = 6$ and $\int_3^5 f(x)\,dx = 4$, then $\int_0^5 (3 + 2f(x))\,dx =$

(A) 10 (B) 20 (C) 23 (D) 35 (E) 50

$$\int_0^5 = 10$$

$$\int_0^5 3\,dx + 2\int_0^5 f(x)\,dx$$

$$3(5-0) + 2(10)$$

78. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?

(A) The change in temperature during the first day

(B) The change in temperature during the 24th hour

(C) The average rate at which the temperature changed during the 24th hour

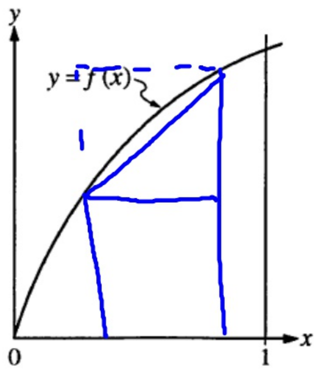
(D) The rate at which the temperature is changing during the first day

(E) The rate at which the temperature is changing at the end of the 24th hour

79. A spherical tank contains 81.637 gallons of water at time $t = 0$ minutes. For the next 6 minutes, water flows out of the tank at the rate of $9\sin\left(\sqrt{t+1}\right)$ gallons per minute. How many gallons of water are in the tank at the end of the 6 minutes?

(A) 36.606 (B) 45.031 (C) 68.858 (D) 77.355 (E) 126.668

$$81.637 - \int_0^6 9\sin\sqrt{t+1}\,dt$$



80. A left Riemann sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x)\,dx$, each using the same number of subintervals. The graph of the function f is shown in the figure above. Which of the sums give an underestimate of the value of $\int_0^1 f(x)\,dx$?

I. Left sum
II. Right sum
III. Trapezoidal sum

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

Riemann $f\uparrow$ $f\downarrow$

Trap CU/CD

81. The first derivative of the function f is given by $f'(x) = x - 4e^{-\sin(2x)}$. How many points of inflection does the graph of f have on the interval $0 < x < 2\pi$?

(A) Three (B) Four (C) Five (D) Six (E) Seven

82. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

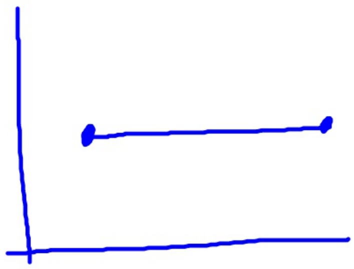
(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.

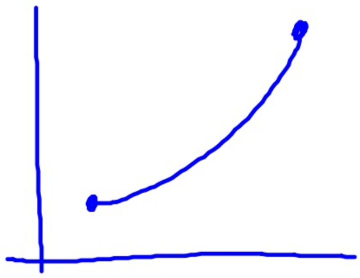
(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$.

(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$.

(E) There is a number c in the open interval (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.





x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

83. The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \le x \le 5$. Which of the following could be the value of $f'(3)$?

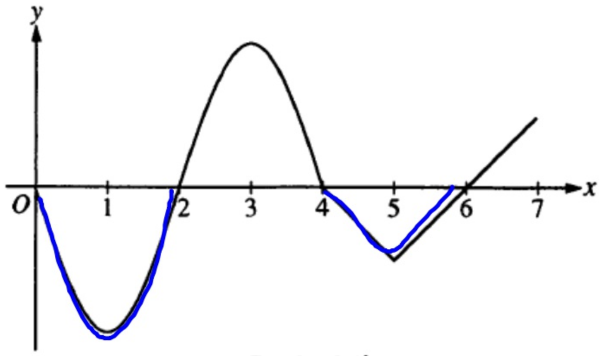
(A) 20 (B) 27.5 (C) 29 $\boxed{\text{(D) 30}}$ (E) 30.5

$f' > 0$ $f'' > 0$

$$\frac{48.05 - 39.2}{.3} = 29.5$$

$$\frac{45 - 39.2}{.2} = 29 \longrightarrow f'_{-}(3) =$$

$$\frac{48.05 - 45}{.1} = 30.5 \longrightarrow f'_{+}(3) =$$



Graph of f'

84. The graph of f', the derivative of the function f, is shown above. On which of the following intervals is f decreasing?

(A) $[2, 4]$ only

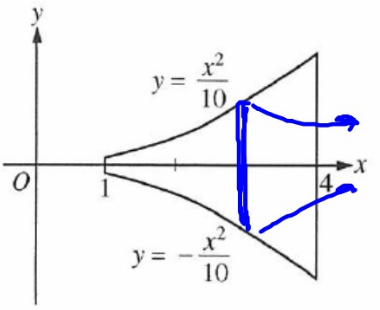
(B) $[3, 5]$ only

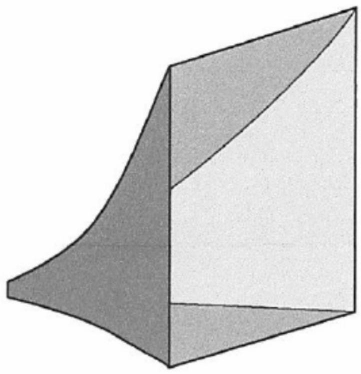
(C) $[0, 1]$ and $[3, 5]$

(D) $[2, 4]$ and $[6, 7]$

(E) $[0, 2]$ and $[4, 6]$

$(0,2) \cup (4,6)$





85. The base of a loudspeaker is determined by the two curves $y = \frac{x^2}{10}$ and $y = -\frac{x^2}{10}$ for $1 \le x \le 4$, as shown in the figure above. For this loudspeaker, the cross sections perpendicular to the x-axis are squares. What is the volume of the loudspeaker, in cubic units?

(A) 2.046 (B) 4.092 (C) 4.200 (D) 8.184 (E) 25.711

$$V = \int_1^4 \left(\frac{x^2}{5}\right)^2 dx$$

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

$f'<0$ $f'<0$ $f'>0$ $f'>0$

86. The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

I. The minimum value of f on $[3, 7]$ ~~is~~ 12.

II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$.

III. $f'(x) > 0$ for $5 < x < 7$.

(A) I only

(B) II only

(C) III only

(D) I and III only

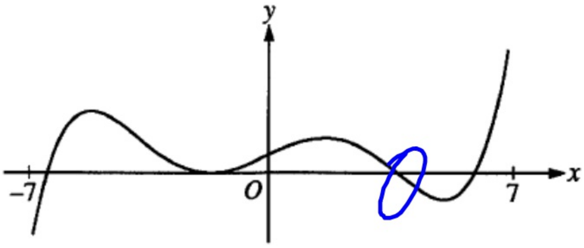
(E) I, II, and III

RT

$f'(x)=0$

3.5
−8000

6.5
−8000



Graph of f'

87. The figure above shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) One (B) Two (C) Three (D) Four (E) Five

88. The rate at which water is sprayed on a field of vegetables is given by $R(t) = 2\sqrt{1 + 5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \le t \le 4$, what is the average rate of water flow, in gallons per minute?

(A) 8.458 (B) 13.395 (C) 14.691 (D) 18.916 (E) 35.833

$$\frac{1}{4}\int_0^4 R(t)\,dt$$

average rate of change in ___

$$\frac{R(4) - R(0)}{4 - 0}$$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	–2	–3	4

89. The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If $h(x) = \underbrace{(2f(x)+3)}\underbrace{(1+g(x))}$, then $h'(1) =$

(A) –28 (B) –16 (C) 40 (D) 44 (E) 47

$$h'(x) = [2f(x)+3][g'(x)] + [1+g(x)][2f'(x)]$$

$$h'(1) =$$

90. The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$.
If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

(A) ~~$g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$~~

(B) ~~$g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$~~

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

3, 8 on f

$f(3) = 8$ $\quad g(8) = 3$

$g'(8) = \frac{1}{f'(3)}$

If (c, d) on $f \rightarrow (f^{-1})'(d) = \frac{1}{f'(c)}$

91. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 5te^{-t} - 1$. At $t = 0$, the particle is at position $x = 1$. What is the total distance traveled by the particle from $t = 0$ to $t = 4$?

(A) 0.366 (B) 0.542 (C) 1.542 (D) 1.821 (E) 2.821

$$\int_0^4 |v(t)|\,dt =$$

$$1 + \int_0^4 v(t)\,dt$$

92. Let f be the function with first derivative defined by $f'(x) = \sin(x^3)$ for $0 \le x \le 2$. At what value of x does f attain its maximum value on the closed interval $0 \le x \le 2$?

(A) 0 (B) 1.162 (C) 1.465 (D) 1.845 (E) 2