82. If f is a continuous function such that $f(2) = 6$, which of the following statements must be true?

(A) $\lim_{x \to 1} f(2x) = 3$

(B) $\lim_{x \to 2} f(2x) = 12$

(C) $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = 6$

(D) $\lim_{x \to 2} f(x^2) = 36$

(E) $\lim_{x \to 2} (f(x))^2 = 36$

$36 = 36$

$\lim_{x \to 2} f(x) = 6$

$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$

83. A particle moves along a straight line with velocity given by $v(t) = 5 + e^{t/3}$ for time $t \geq 0$. What is the acceleration of the particle at time $t = 4$?

(A) 0.422 (B) 0.698 (C) 1.265 (D) 8.794 (E) 28.381

$v'(4)$

y1 = v(t)
y2 = d(y1(x),x)

y2(4)

84. A home uses fuel oil at the rate $r(t) = 10 + 8\sin\left(\frac{t}{60}\right)$ gallons per day, where t is the number of days from the beginning of the heating season. To the nearest gallon, what is the total amount of fuel oil used from $t = 0$ to $t = 60$ days?

(A) 7 gal (B) 14 gal (C) 600 gal (D) 821 gal (E) 1004 gal

$$\int_0^{60} r(t)\,dt = 820.655$$

y1 = r(t)

∫(y1(x), x, 0, 60)

85. The function f is defined on the open interval $0.4 < x < 2.4$ and has first derivative f' given by $f'(x) = \sin\left(x^2\right)$. Which of the following statements are true?

I. f has a relative maximum on the interval $0.4 < x < 2.4$. A → B

II. f has a relative minimum on the interval $0.4 < x < 2.4$. B → A

III. The graph of f has two points of inflection on the interval $0.4 < x < 2.4$. nel ext

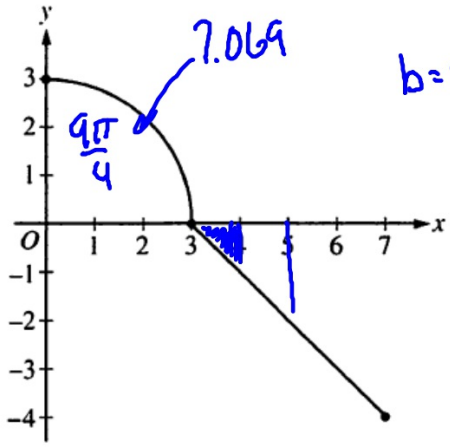
(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only



Graph of f

b=5 $\frac{1}{2}(2)^2 = 2$

b=6 $\frac{1}{2}(3)(3) = 4.5$

b=7 $\frac{1}{2}(4)(4) = 8$

86. The graph of the function f, which has a domain of $[0, 7]$, is shown in the figure above. The graph consists of a quarter circle of radius 3 and a segment with slope -1. Let b be a positive number such that $\int_0^b f(x)\,dx = 0$. What is the value of b?

(A) 3.760 $-3)^2 \div 2$

(B) 5.548

(C) 5.659

(D) 6.760

(E) There is no such value of b.

87. The first derivative of the function g is given by $g'(x) = \cos(\pi x^2)$ for $-0.5 < x < 1.5$. On which of the following intervals is g decreasing?

$g'(x) < 0$

(A) $-0.5 < x < 0$

(B) $0 < x < 1$

(C) $0.707 < x < 1.225$

(D) $1.225 < x < 1.414$

(E) $1.414 < x < 1.5$

88. The height above the ground of a passenger on a Ferris wheel t minutes after the ride begins is modeled by the differentiable function H, where $H(t)$ is measured in meters. Which of the following is an interpretation of the statement $H'(7.5) = 15.708$?

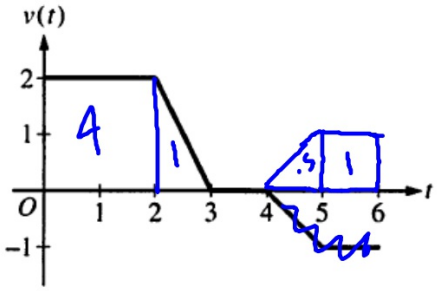
(A) The Ferris wheel is turning at a rate of 15.708 meters per minute when the passenger is 7.5 meters above the ground.

(B) The Ferris wheel is turning at a rate of 15.708 meters per minute 7.5 minutes after the ride begins.

(C) The passenger's height above the ground is increasing by 15.708 meters per minute when the passenger is 7.5 meters above the ground.

(D) The passenger's height above the ground is increasing by 15.708 meters per minute 7.5 minutes after the ride begins.

(E) The passenger is 15.708 meters above the ground 7.5 minutes after the ride begins.



89. A particle moves along a straight line for 6 seconds so that its velocity, in centimeters per second, is modeled by the graph shown. During the time interval $0 \le t \le 6$, what is the total distance the particle travels?

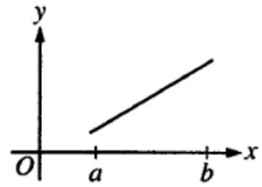
(A) 2 cm (B) 3.5 cm (C) 4 cm (D) 6.5 cm (E) 8.5 cm

$$\int_0^6 |v(t)|\,dt$$

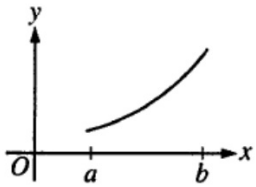
90. Let f be a twice-differentiable function on the open interval (a, b). If $f'(x) > 0$ on (a, b) and $f''(x) < 0$ on (a, b), which of the following could be the graph of f?

f↑ CD

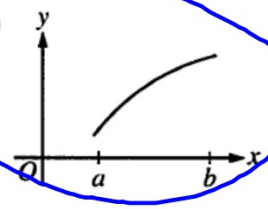
(A)



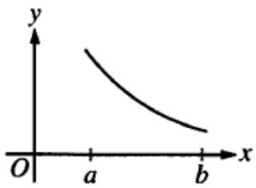
(B)



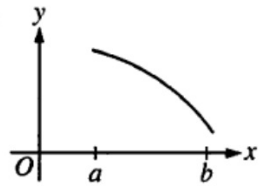
(C)

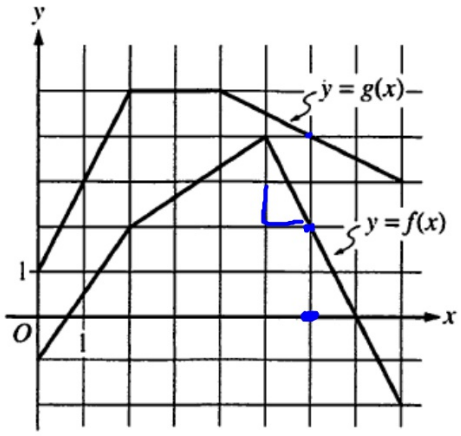


(D)



(E)





$\frac{-1}{2}$

$\frac{-2}{1}$

91. The graphs of f and g are shown above. If $h(x) = f(x)g(x)$, then $h'(6) =$

(A) -9 (B) -7 (C) 1 (D) 7 (E) 9

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$h'(6) = f(6)g'(6) + g(6)f'(6)$

$(2)\left(-\frac{1}{2}\right) + (4)(-2)$

$-1 - 8$

92. In the xy-plane, the graph of the twice-differentiable function $y = f(x)$ is concave up on the open interval $(0, 2)$ and is tangent to the line $y = 3x - 2$ at $x = 1$. Which of the following statements must be true about the derivative of f?

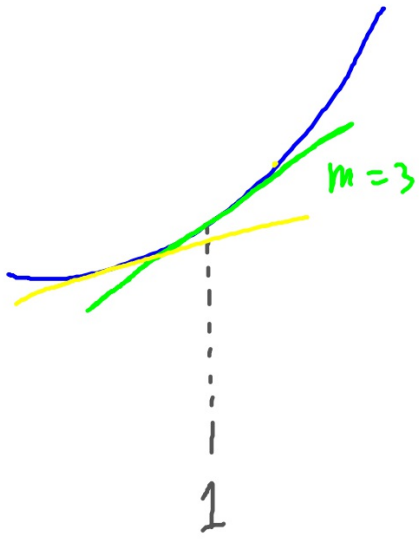
(A) $f'(x) \leq 3$ on the interval $(0.9, 1)$.

(B) $f'(x) \geq 3$ on the interval $(0.9, 1)$.

(C) $f'(x) < 0$ on the interval $(0.9, 1.1)$.

(D) $f'(x) > 0$ on the interval $(0.9, 1.1)$.

(E) $f'(x)$ is constant on the interval $(0.9, 1.1)$.



1. A company produces and sells chili powder. The company's weekly profit on the sale of x kilograms of chili powder is modeled by the function P given by $P(x) = 48x + 1.4x^2 - 0.05x^{2.8} - 270$, where $P(x)$ is in dollars and $0 \le x \le 80$.

   (a) Find the rate, in dollars per kilogram, at which the company's weekly profit is changing when it sells 32 kilograms of chili powder. Is the company's weekly profit increasing or decreasing when it sells 32 kilograms of chili powder? Give a reason for your answer.

$P'(x) = -.14x^{1.8} + 2.8x + 48$

$P'(32) = 65.920$

$\therefore$ profit changing at 65.920 dol/week.

Profit is increasing because $P'(32) > 0$.

A company produces and sells chili powder. The company's weekly profit on the sale of x kilograms of chili powder is modeled by the function P given by $P(x) = 48x + 1.4x^2 - 0.05x^{2.8} - 270$, where $P(x)$ is in dollars and $0 \le x \le 80$.

(b) How many kilograms of chili powder must the company sell to maximize its weekly profit? Justify your answer.

$P'(x) = -.14x^{1.8} + 2.8x + 48$

$P'(x) = 0 \rightarrow x = 58.358$

$P(0) = -270$

$P(80) = 1873.319$

$P(58.358) = 2893.035$

Since P is continuous on $[0, 80]$ kg EUT the weekly profit is a maximum at $x = 58.358$ kg.

(c) The company plans to have a one-day sale on chili powder. Management estimates that t hours after the company store opens, chili powder will sell at a rate modeled by the function S given by $S(t) = 2 + \cos\left(\frac{\pi}{10}t^2\right)$ kilograms per hour. Based on this model, estimate the amount of chili powder, in kilograms, that will be sold during the first 5 hours of the sale.

$$\int_0^5 S(t)\,dt = 11.433$$

$\therefore 11.433$ kg

(d) Using the function S from part (c), find the value of $S'(3)$. Interpret the meaning of this value in the context of the problem.

$$S'(t) = \frac{-\pi t \sin \frac{\pi t^2}{10}}{5}$$

$$S'(3) = -.582$$

Rate at which sales are made is decreasing at .582 kg/(in/hr.) at $t = 3$ hrs.