

10. $\int (e^x + e) dx =$

(A) $e^x + C$

(B) $2e^x + C$

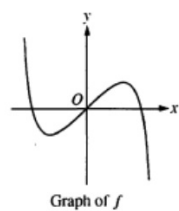
(C) $e^x + e + C$

(D) $e^{x+1} + ex + C$

(E) $e^x + ex + C$

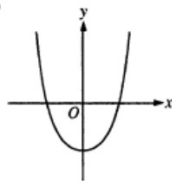
$$e^x + ex + C$$

$$\begin{aligned} \int [e^x + 5] dx \\ = e^x + 5x \end{aligned}$$

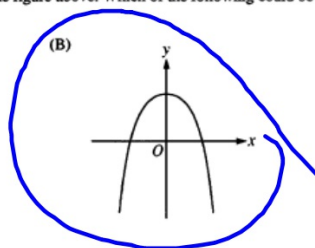


11. The graph of the function f is shown in the figure above. Which of the following could be the graph of f' , the derivative of f ?

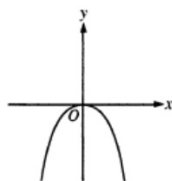
(A)



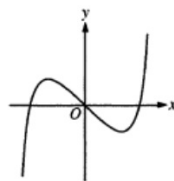
(B)



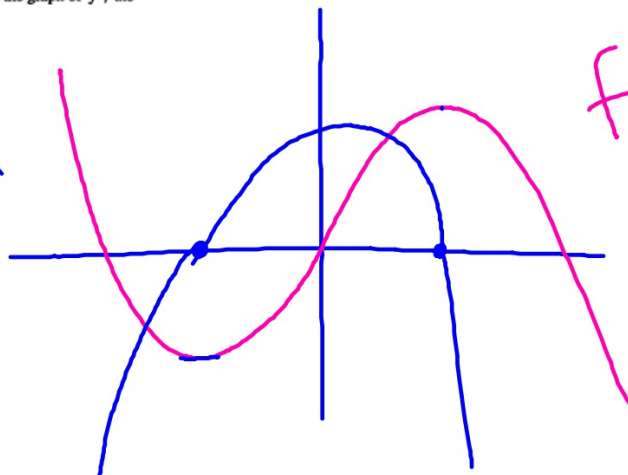
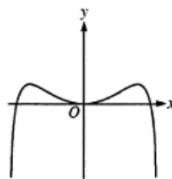
(C)



(D)

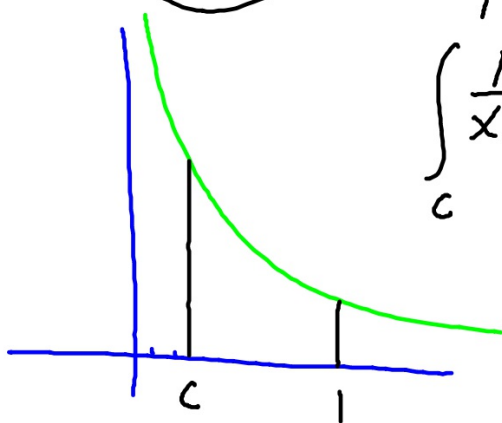


(E)



12. If $0 < c < 1$, what is the area of the region enclosed by the graphs of $y = 0$, $y = \frac{1}{x}$, $x = c$, and $x = 1$?

- (A) $\ln(1-c)$ (B) $\ln\left(\frac{1}{c}\right)$ (C) $\ln c$ (D) $\frac{1}{c^2} - 1$ (E) $1 - \frac{1}{c^2}$



$$\int_c^1 \frac{1}{x} dx = \ln|x| \Big|_c^1 = \cancel{\ln 1} - \ln c$$

$$= -\ln c$$

$$= \ln c^{-1}$$

$$= \ln \frac{1}{c}$$

$$\ln_e 1 = ?$$

13. $\frac{d}{dx}(\tan^{-1}x + 2\sqrt{x}) =$

(A) $-\frac{1}{\sin^2 x} + \frac{1}{\sqrt{x}}$

(B) $\frac{1}{\sqrt{1-x^2}} - 4\sqrt[3]{x}$

(C) $\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x}}$

(D) $\frac{1}{1+x^2} - 4\sqrt[3]{x}$

(E) $\frac{1}{1+x^2} + \frac{1}{\sqrt{x}}$

$$\frac{1}{1+x^2} + 2\frac{1}{2\sqrt{x}}$$

$$D_x[\tan^{-1}x] = \frac{x'}{1+x^2}$$

14. If $y = f(x)$ is a solution to the differential equation $\frac{dy}{dx} = e^{x^2}$ with the initial condition $f(0) = 2$, which of the following is true?

(A) $f(x) = 1 + e^{x^2}$

(B) $f(x) = 2xe^{x^2}$

(C) $f(x) = \int_1^x e^{t^2} dt$ ✓

(D) $f(x) = 2 + \int_0^x e^{t^2} dt$ ✓

(E) $f(x) = 2 + \int_2^x e^{t^2} dt$ ✓

$$dy = e^{x^2} dx$$

$$y = \boxed{}$$

$$\int x^2$$

$$\boxed{\frac{1}{3}x^3}$$

$$f(0) = \int_1^2 e^{t^2} dt$$

$$\frac{d}{dx} \int_1^x e^{t^2} dt = e^{x^2}$$

$$\underline{f(0)} = 2 + \int_0^0 = \underline{2}$$

15. A function $f(t)$ gives the rate of evaporation of water, in liters per hour, from a pond, where t is measured in hours since 12 noon. Which of the following gives the meaning of $\int_4^{10} f(t) dt$ in the context described?

- (A) The total volume of water, in liters, that evaporated from the pond during the first 10 hours after 12 noon
- (B) The total volume of water, in liters, that evaporated from the pond between 4 P.M. and 10 P.M.
- (C) The net change in the rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.
- (D) The average rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.
- (E) The average rate of change in the rate of evaporation, in liters per hour per hour, from the pond between 4 P.M. and 10 P.M.

Total amt of H_2O in L evap
from 4pm to 10pm

16. The first derivative of the function f is given by $f'(x) = 3x^4 - 12x^3$. What are the x -coordinates of the points of inflection of the graph of f ?

(A) $x = 3$ only

(B) $x = 4$ only

(C) $x = 0$ and $x = 2$

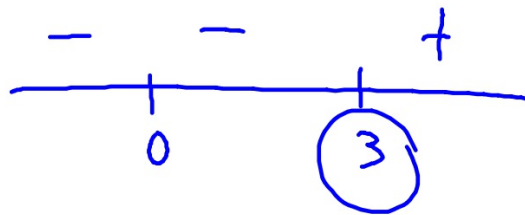
(D) $x = 0$ and $x = 3$

(E) $x = 0$ and $x = 4$

$$f''(x) = 12x^3 - 36x^2$$

$$12x^2(x - 3)$$

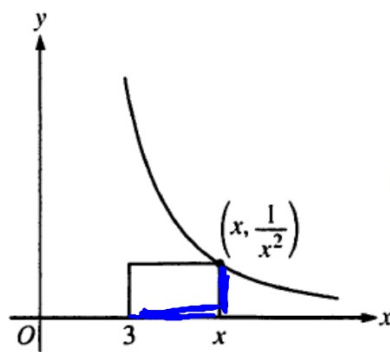
$$x = 0 \text{ or } x = 3$$



17. Let f be the function defined by $f(x) = \frac{1}{x}$. What is the average value of f on the interval $[4, 6]$?

- (A) $-\frac{1}{24}$ (B) $\frac{5}{24}$ (C) $\frac{1}{2} \ln \frac{3}{2}$ (D) $\ln \frac{3}{2}$ (E) $\frac{1}{2} \ln 2$

$$\begin{aligned} \frac{1}{2} \int_4^6 \frac{1}{x} dx &= \frac{1}{2} \left[\ln |x|_4^6 \right] \\ &= \frac{1}{2} \left[\ln 6 - \ln 4 \right] \\ &= \frac{1}{2} \ln \frac{3}{2} \end{aligned}$$



$$A(x) = \frac{(x-3)}{x^2}$$

18. The points $(3, 0)$, $(x, 0)$, $(x, \frac{1}{x^2})$, and $(3, \frac{1}{x^2})$ are the vertices of a rectangle, where $x \geq 3$, as shown in the figure above. For what value of x does the rectangle have a maximum area?

(A) 3

(B) 4

(C) 6

(D) 9

(E) There is no such value of x .

$$A'(x) = \frac{x^2 - (x-3)(2x)}{x^4}$$

$$x^2 - 2x^2 + 6x$$

$$6x - x^2 = 0$$

$$x(6-x) = 0$$

$$x = 0 \text{ or } x = 6$$

19. What are all values of x for which $\int_x^2 t^3 dt$ is equal to 0?

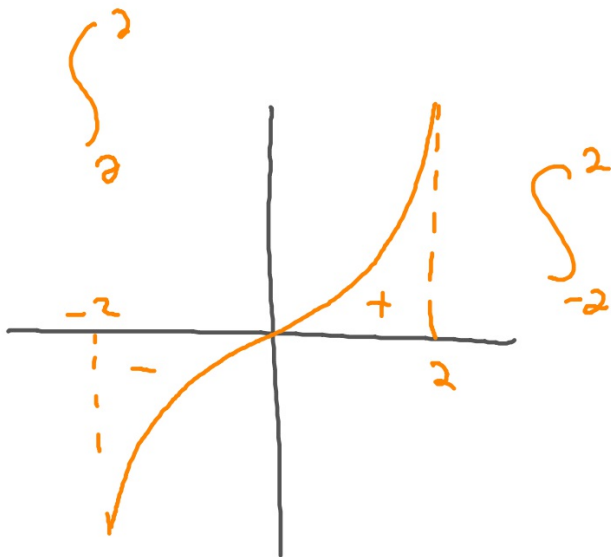
(A) -2 only

(B) 0 only

(C) 2 only

(D) -2 and 2 only

(E) -2, 0, and 2



20. Let h be the function defined by $h(x) = \int_{\pi/4}^x \sin^2 t \, dt$. Which of the following is an equation for the line tangent to the graph of h at the point where $x = \frac{\pi}{4}$?

(A) $y = \frac{1}{2}$

(B) $y = \sqrt{2}x$

(C) $y = x - \frac{\pi}{4}$

(D) $y = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

(E) $y = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$$h\left(\frac{\pi}{4}\right) = 0$$

$$\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4}$$

$$h'(x) = \sin^2 x$$

$$h'\left(\frac{\pi}{4}\right) = \left(\sin \frac{\pi}{4}\right)^2 = \frac{1}{2}$$

$$y = 0 + \frac{1}{2}\left(x - \frac{\pi}{4}\right)$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

MVT

x	$f(x)$
-1	-30
0	-2
3	10
5	18

21. The table above gives selected values for a twice-differentiable function f . Which of the following must be true?

- (A) f has no critical points in the interval $-1 < x < 5$.
- (B) $f'(x) = 8$ for some value of x in the interval $-1 < x < 5$.
- (C) $f'(x) > 0$ for all values of x in the interval $-1 < x < 5$.
- (D) $f''(x) < 0$ for all values of x in the interval $-1 < x < 5$.
- (E) The graph of f has no points of inflection in the interval $-1 < x < 5$.

$$\frac{f(5) - f(-1)}{5 - (-1)} = \frac{18 - (-30)}{6} = 8$$

$$\text{by MVT } f'(c) = 8$$

22. A particle moves along the x -axis so that at time $t \geq 0$, the acceleration of the particle is $a(t) = 15\sqrt{t}$. The position of the particle is 10 when $t = 0$, and the position of the particle is 20 when $t = 1$. What is the velocity of the particle at time $t = 0$?

(A) -14 (B) 0 (C) 5 (D) 6 (E) 10

$$a(t) = 15t^{1/2}$$

$$v(t) = 10t^{3/2} + C$$

$$s(t) = 4t^{5/2} + Ct + D$$

$$10 = D$$

$$15 \cdot \frac{2}{3} t^{3/2} \quad 10 \frac{2t^{5/2}}{5}$$

$$s(t) = 4t^{5/2} + Ct + 10$$

$$20 = 4 + C + 10$$

$$6 = C$$

$$v(t) = 10t^{3/2} + 6$$

$$v(0) = 6$$