

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F . The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6}$$

The water temperature is increasing
at about $1.017^\circ\text{F}/\text{min}$ at $t = 12\text{min}$.

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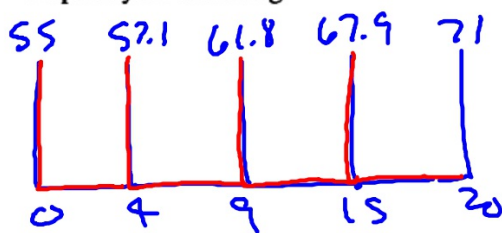
- (b) Use the data in the table to evaluate $\int_0^{20} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t) dt$ in the context of this problem.

$$\int_0^{20} W'(t) dt = W(20) - W(0) = 71 - 55 = 16$$

The water temperature has warmed
by 16°F from $t=0$ min to $t=20$ min.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

- (c) For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.



$$\frac{1}{20} \int_0^{20} W(t) dt \approx \frac{1}{20} [55(4) + 57.1(5) + 61.8(6) + 67.9(5)]$$

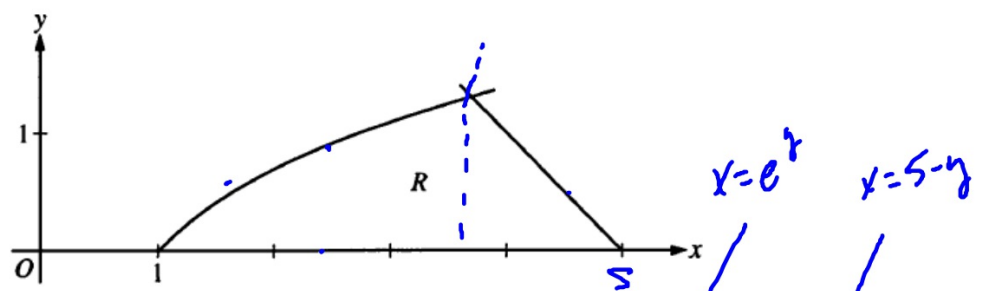
This approximation is an underestimate because $W(t)$ is strictly increasing and we used a left Riemann sum.

- (d) For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$$71 + \int_{20}^{25} W'(t) dt = 73.043$$

$\therefore 73.043^{\circ}\text{F}$

$$\begin{array}{r} 2.043 \\ \hline 71 \end{array}$$



2. Let R be the region in the first quadrant bounded by the x -axis and the graphs of $y = \ln x$ and $y = 5 - x$ as shown in the figure above.

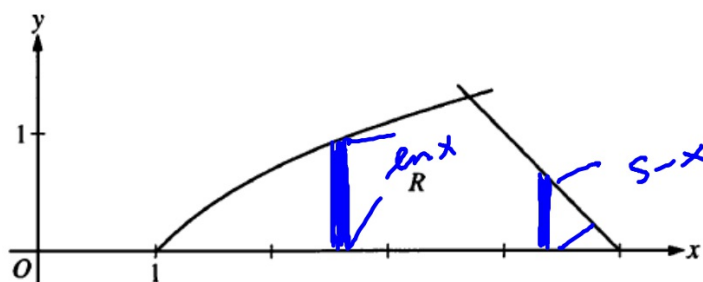
(a) Find the area of R .

1 sect
 $\ln x = 5 - x$
 $x = 3.693$
 $y = 1.307$

$$A = \int_1^{3.693} \ln x \, dx + \int_{3.693}^5 (5 - x) \, dx = 2.986$$

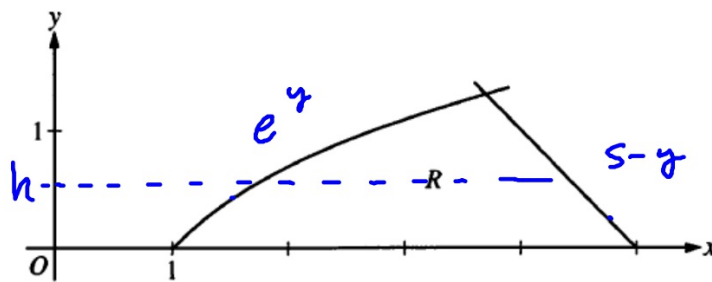
OR

$$A = \int_0^{1.307} (5 - y - e^y) \, dy = 2.986$$



- (b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

$$V = \int_1^{3.693} (\ln x)^2 dx + \int_{3.693}^5 (5-x)^2 dx$$



- (c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .

$$\int_0^k (5 - y - e^y) dy = \frac{2.986}{2}$$

or

$$\int_0^k (5 - y - e^y) dy = \int_k^{1.037} (5 - y - e^y) dy$$

13. Let f be a differentiable function such that $f(0) = -5$ and $f'(x) \leq 3$ for all x . Of the following, which is not a possible value for $f(2)$?

(A) -10

(B) -5

(C) 0

(D) 1

(E) 2

$$\frac{f(2) - f(0)}{2 - 0} \leq 3$$

$$f(2) + 5 \leq 6$$

$$f(2) \leq 1$$

$$\begin{array}{l} f(2) \\ f(0) \\ f'(x) \end{array}$$

$$f(x) = \begin{cases} x + b & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

Cont. test
Diff. test

14. Let f be the function given above. What are all values of a and b for which f is differentiable at $x = 1$?

(A) $a = \frac{1}{2}$ and $b = -\frac{1}{2}$

(B) $a = \frac{1}{2}$ and $b = \frac{3}{2}$

(C) $a = \frac{1}{2}$ and b is any real number

(D) $a = b + 1$, where b is any real number

(E) There are no such values of a and b .

$$f'(x) = \begin{cases} 1 & x < 1 \\ 2ax & x > 1 \end{cases}$$

$$f'_+(1) = 2a$$

$$f'_-(1) = 1$$

$$2a = 1$$

$$a = \frac{1}{2}$$

$$f(1) = 1 + b$$

$$\lim_{x \rightarrow 1^+} f(x) = a$$

$$\lim_{x \rightarrow 1^-} f(x) = 1 + b$$

$$1 + b = a$$

$$1 + b = \frac{1}{2} \quad b = -\frac{1}{2}$$

$f(3)$	$g(3)$	$f'(3)$	$g'(3)$
-1	2	5	-2

15. The table above gives values for the functions f and g and their derivatives at $x = 3$. Let k be the function given by $k(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$. What is the value of $k'(3)$?

(A) $-\frac{5}{2}$ (B) -2 (C) 2 (D) 3 (E) 8

$$k'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\begin{aligned} k'(3) &= \frac{(2)(5) - (-1)(-2)}{4} \\ &= \frac{10 - 2}{4} \\ &= 2 \end{aligned}$$

16. If $y = 5x\sqrt{x^2 + 1}$, then $\frac{dy}{dx}$ at $x = 3$ is

(A) $\frac{5}{2\sqrt{10}}$

(B) $\frac{15}{\sqrt{10}}$

(C) $\frac{15}{2\sqrt{10}} + 5\sqrt{10}$

(D) $\frac{45}{\sqrt{10}} + 5\sqrt{10}$

(E) $\frac{45}{\sqrt{10}} + 15\sqrt{10}$

$$\frac{dy}{dx} = (5x) \frac{\cancel{2x}}{\sqrt{x^2+1}} + 5\sqrt{x^2+1}$$

$$(15) \frac{3}{\sqrt{10}} + 5\sqrt{10}$$

$$\frac{45}{\sqrt{10}} + 5\sqrt{10}$$

17. If $\lim_{h \rightarrow 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a ?

- (A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{1}{2}$ (E) 2

$$f(x) = \sin^{-1} x$$

$$f'(a) = 2$$

$$\sin^{-1} x \Rightarrow \frac{x'}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-a^2}} = 2$$

$$2\sqrt{1-a^2} = 1$$

$$\sqrt{1-a^2} = \frac{1}{2}$$

$$1-a^2 = \frac{1}{4}$$

$$a^2 = \frac{3}{4}$$

$$a = \pm \frac{\sqrt{3}}{2}$$

18. If $\ln(2x+y) = x+1$, then $\frac{dy}{dx} =$

(A) -2

(B) $2x+y-2$

(C) $2x+y$

(D) $4x+2y-2$

(E) $y - \frac{y}{x}$

$$\frac{2 + \frac{dy}{dx}}{2x+y} = 1$$

$$2 + \frac{dy}{dx} = 2x+y$$

$$\frac{dy}{dx} = 2x+y-2$$

$$\ln \cancel{A} \rightarrow \frac{\cancel{A}'}{\cancel{A}}$$