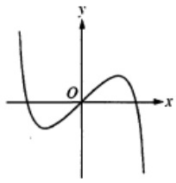
10. $\int (e^x + e)\, dx =$

(A) $e^x + C$ (B) $2e^x + C$ (C) $e^x + e + C$ (D) $e^{x+1} + ex + C$ (E) $e^x + ex + C$

$e^x + ex + c$

$\int (e^x + 5)\, dx$

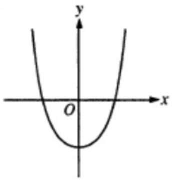
$e^x + 5x + c$



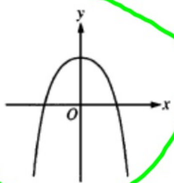
Graph of f

11. The graph of the function f is shown in the figure above. Which of the following could be the graph of f', the derivative of f?

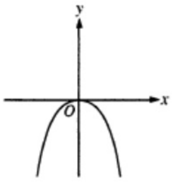
(A)



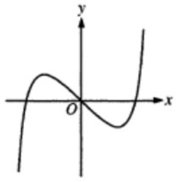
(B)



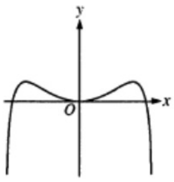
(C)

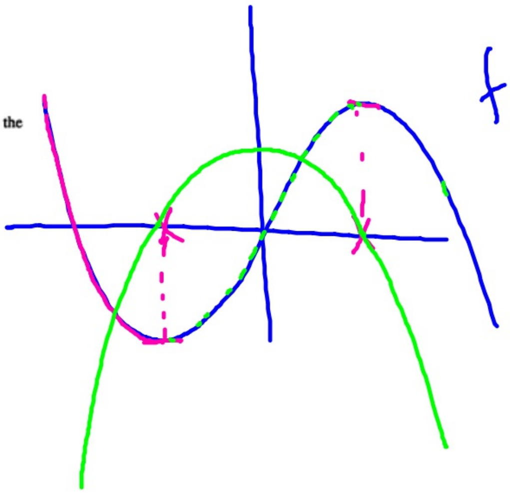


(D)



(E)



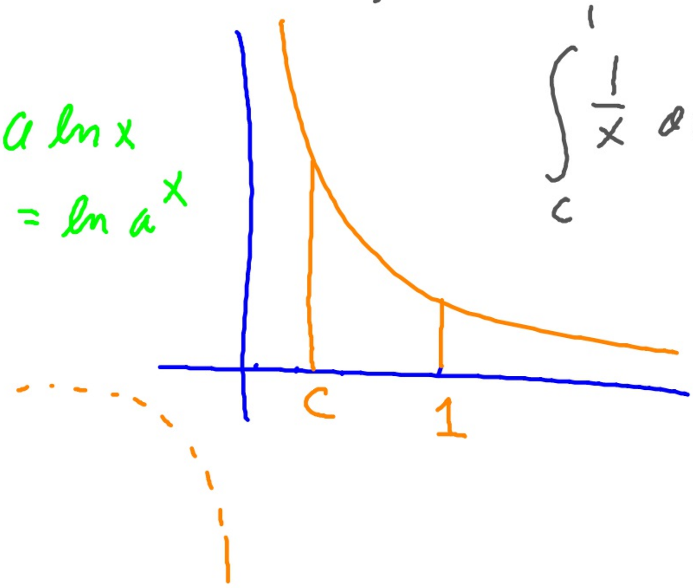


12. If $0 < c < 1$, what is the area of the region enclosed by the graphs of $y = 0$, $y = \frac{1}{x}$, $x = c$, and $x = 1$?

(A) $\ln(1 - c)$ (B) $\ln\left(\frac{1}{c}\right)$ (C) $\ln c$ (D) $\frac{1}{c^2} - 1$ (E) $1 - \frac{1}{c^2}$

$a \ln x$

$= \ln a^x$



$$\int_c^1 \frac{1}{x}\,dx = \ln|x|\Big|_c^1$$

$$= \ln 1 - \ln c$$

$$= -\ln c$$

$$= \ln c^{-1}$$

$$= \ln \frac{1}{c}$$

13. $\frac{d}{dx}\left(\tan^{-1} x + 2\sqrt{x}\right) =$

(A) $-\frac{1}{\sin^2 x} + \frac{1}{\sqrt{x}}$

(B) $\frac{1}{\sqrt{1-x^2}} - 4\sqrt[3]{x}$

(C) $\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x}}$

(D) $\frac{1}{1+x^2} - 4\sqrt[3]{x}$

(E) $\frac{1}{1+x^2} + \frac{1}{\sqrt{x}}$

$$\frac{1}{1+x^2} + \cancel{2}\frac{1}{\cancel{2}\sqrt{x}}$$

$$D_x[\tan^{-1} A] = \frac{A'}{1+A^2}$$

14. If $y = f(x)$ is a solution to the differential equation $\frac{dy}{dx} = e^{x^2}$ with the initial condition $f(0) = 2$, which of the following is true?

(A) $f(x) = 1 + e^{x^2}$

(B) $f(x) = 2xe^{x^2}$

(C) $f(x) = \int_1^x e^{t^2}\,dt$ ✓

(D) $f(x) = 2 + \int_0^x e^{t^2}\,dt$ ✓

(E) $f(x) = 2 + \int_2^x e^{t^2}\,dt$ ✓

$dy = e^{x^2}\,dx$

$= \boxed{}$

$f(0) = \int_1^0 e^{t^2}\,dt$

$\frac{d}{dx}\int_1^x e^{t^2}\,dt = e^{x^2}$

$f(0) = 2 + \int_0^0 e^{t^2}\,dt = 2$

15. A function $f(t)$ gives the rate of evaporation of water, in liters per hour, from a pond, where t is measured in hours since 12 noon. Which of the following gives the meaning of $\boxed{\int_4^{10} f(t)\,dt}$ in the context described?

(A) The total volume of water, in liters, that evaporated from the pond during the first 10 hours after 12 noon

(B) The total volume of water, in liters, that evaporated from the pond between 4 P.M. and 10 P.M.

(C) The net change in the rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

(D) The average rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

(E) The average rate of change in the rate of evaporation, in liters per hour per hour, from the pond between 4 P.M. and 10 P.M.

total # of L of H_2O
evap fm 4pm
to 10pm

16. The first derivative of the function f is given by $f'(x) = 3x^4 - 12x^3$. What are the x-coordinates of the points of inflection of the graph of f ?

(A) $x = 3$ only

(B) $x = 4$ only

(C) $x = 0$ and $x = 2$

(D) $x = 0$ and $x = 3$

(E) $x = 0$ and $x = 4$

$$f''(x) = 12x^3 - 36x^2$$

$$12x^2(x-3) = 0$$

$$x = 0 \quad \text{or} \quad x = 3$$

$-$ $\quad$ $-$ $\quad$ $+$

0 $\quad$ 3

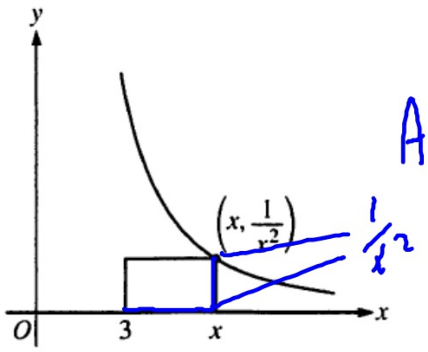
17. Let f be the function defined by $f(x) = \frac{1}{x}$. What is the average value of f on the interval $[4, 6]$?

(A) $-\frac{1}{24}$ (B) $\frac{5}{24}$ (C) $\frac{1}{2}\ln\frac{3}{2}$ (D) $\ln\frac{3}{2}$ (E) $\frac{1}{2}\ln 2$

$$\frac{1}{2}\int_4^6 \frac{1}{x}\,dx = \frac{1}{2}\Big[\ln|x|\Big]_4^6$$

$$= \frac{1}{2}\left[\ln 6 - \ln 4\right]$$

$$= \frac{1}{2}\ln\frac{3}{2}$$



$$A(x) = (x-3)\frac{1}{x^2}$$

$$= \frac{x-3}{x^2}$$

18. The points $(3, 0)$, $(x, 0)$, $\left(x, \frac{1}{x^2}\right)$, and $\left(3, \frac{1}{x^2}\right)$ are the vertices of a rectangle, where $x \geq 3$, as shown in the figure above. For what value of x does the rectangle have a maximum area?

(A) 3

(B) 4

(C) 6

(D) 9

(E) There is no such value of x.

$$A'(x) = \frac{x^2 - (x-3)(2x)}{x^4}$$

$$= \frac{x^2 - 2x^2 + 6x}{x^4}$$

$$6x - x^2 = 0$$

$$x(6-x) = 0$$

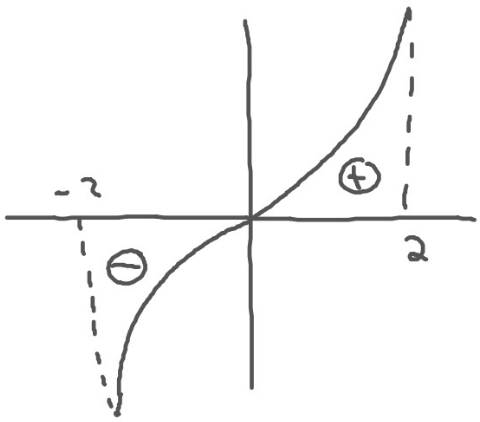
$$\cancel{x = 0} \text{ or } x = 6$$

19. What are all values of x for which $\int_x^2 t^3\,dt$ is equal to 0 ?

(A) -2 only (B) 0 only (C) 2 only (D) -2 and 2 only (E) -2, 0, and 2

$x = 2$

$x = -2$



20. Let h be the function defined by $h(x) = \int_{\pi/4}^{x} \sin^2 t\, dt$. Which of the following is an equation for the line tangent to the graph of h at the point where $x = \frac{\pi}{4}$?

$\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4}$

(A) $y = \frac{1}{2}$

(B) $y = \sqrt{2}x$

(C) $y = x - \frac{\pi}{4}$

(D) $y = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

(E) $y = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$h\left(\frac{\pi}{4}\right) = 0$

$h'(x) = \sin^2 x$

$h'\left(\frac{\pi}{4}\right) = \left(\sin \frac{\pi}{4}\right)^2 = \frac{1}{2}$

$y = 0 + \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

x	$f(x)$
–1	–30
0	–2
3	10
5	18

21. The table above gives selected values for a twice-differentiable function f. Which of the following must be true?

(A) f has no critical points in the interval $-1 < x < 5$.

(B) $f'(x) = 8$ for some value of x in the interval $-1 < x < 5$.

(C) $f'(x) > 0$ for all values of x in the interval $-1 < x < 5$.

(D) $f''(x) < 0$ for all values of x in the interval $-1 < x < 5$.

(E) The graph of f has no points of inflection in the interval $-1 < x < 5$.

$$\frac{f(b)-f(a)}{b-a} = \frac{f(5)-f(-1)}{5--1} = \frac{18--30}{6} = 8$$

$$f'(c) = \frac{f(b)-f(c)}{b-a}$$

22. A particle moves along the x-axis so that at time $t \geq 0$, the acceleration of the particle is $a(t) = 15\sqrt{t}$. The position of the particle is 10 when $t = 0$, and the position of the particle is 20 when $t = 1$. What is the velocity of the particle at time $t = 0$?

(A) -14 (B) 0 (C) 5 (D) 6 (E) 10

$a(t) = 15t^{1/2}$

$v(t) = 10t^{3/2} + C$

$s(t) = 4t^{5/2} + C\cancel{t} + D$

$10 = 0 + 0 + D$

$15 \frac{2}{3} t^{3/2}$ $10 \frac{2}{5} t^{5/2}$

$s(t) = 4t^{5/2} + Ct + 10$

$20 = 4 + C + 10$

$6 = C$

$v(t) = 10t^{3/2} + 6$

$v(0) = 6$