t (weeks)	0	3	6	10	12
$G(t)$ (games per week)	160	450	900	2100	2400

2. A store tracks the sales of one of its popular board games over a 12-week period. The rate at which games are being sold is modeled by the differentiable function G, where $G(t)$ is measured in games per week and t is measured in weeks for $0 \le t \le 12$. Values of $G(t)$ are given in the table above for selected values of t.

(a) Approximate the value of $G'(8)$ using the data in the table. Show the computations that lead to your answer.

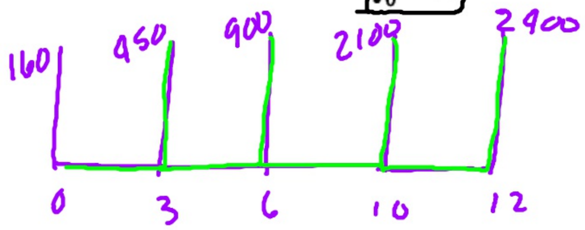
$$G'(8) \approx \frac{G(10) - G(6)}{10 - 6} = \frac{2100 - 900}{4} = 300$$

$\therefore$ 300 games/week^2

$\frac{g}{w} \to g$

t (weeks)	0	3	6	10	12
$G(t)$ (games per week)	160	450	900	2100	2400

(b) Approximate the value of $\int_0^{12} G(t)\,dt$ using a right Riemann sum with the four subintervals indicated by the table. Explain the meaning of $\boxed{\int_0^{12} G(t)}\,dt$ in the context of this problem.



$$\int_0^{12} G(t)\,dt \approx (450)(3) + (900)(3) + (2100)(4) + (2400)(2)$$
$$= 17250$$

$\int_0^{12} G(t)\,dt$ is the total number of games sold (in games) from $t = 0$ weeks to $t = 12$ weeks.

(c) One salesperson believes that, starting with 2400 games per week at time $t = 12$, the rate at which games will be sold will increase at a constant rate of 100 games per week per week. Based on this model, how many total games will be sold in the 8 weeks between time $t = 12$ and $t = 20$?

$$R(t) = 2400 + 100(t-12)$$

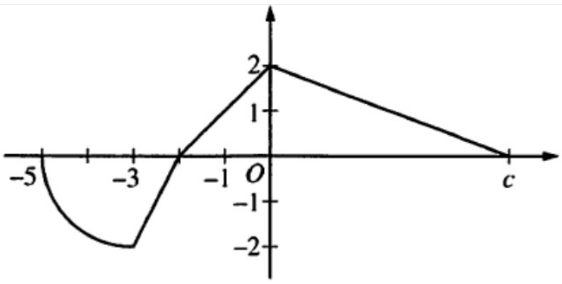
$$\int_{12}^{20} R(t)\,dt = 22400$$

$\therefore$ 22400 games.

(d) Another salesperson believes the best model for the rate at which games will be sold in the 8 weeks between time $t = 12$ and $t = 20$ is $\boxed{M(t) = 2400e^{-0.01(t-12)^2}}$ games per week. Based on this model, how many total games, to the nearest whole number, will be sold during this period?

$$\int_{12}^{20} M(t)\,dt = 15784.077$$

$\therefore$ 15784 games.

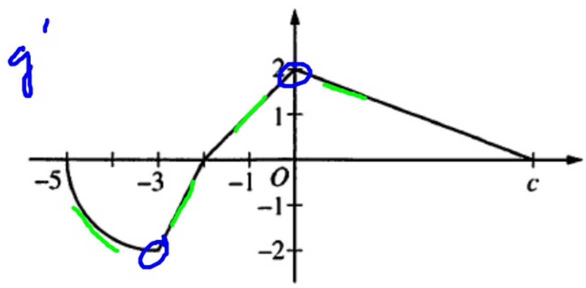


Graph of f

3. The function f is defined on the interval $-5 \le x \le c$, where $c > 0$ and $f(c) = 0$. The graph of f, which consists of three line segments and a quarter of a circle with center $(-3, 0)$ and radius 2, is shown in the figure above.

(a) Find the average rate of change of f over the interval $[-5, 0]$. Show the computations that lead to your answer.

$$\frac{f(0) - f(-5)}{0 - -5} = \frac{2 - 0}{5} = \frac{2}{5}$$



Graph of f

(b) For $-5 \le x \le c$, let g be the function defined by $g(x) = \int_{-1}^{x} f(t)\,dt$. Find the x-coordinate of each point of inflection of the graph of g. Justify your answer.

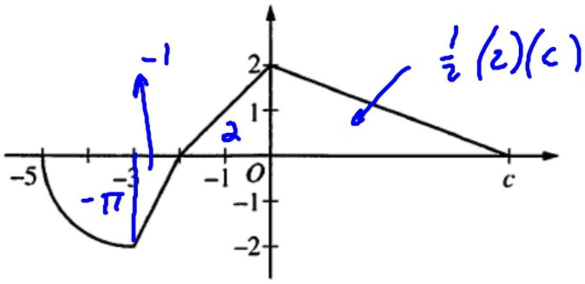
$g'(x) = f(x)$

$g''(x) = f'(x)$

Possible inflection points at $x = -3$ and $x = 0$.

g has inflection point at $x = -3$ because $g''(x) < 0$ on $(-5, -3)$ and $g''(x) > 0$ on $(-3, 0)$.

g has inflection point at $x = 0$ because $g''(x) > 0$ on $(-3, 0)$ and $g''(x) < 0$ on $(0, c)$



Graph of f

$(-5, c)$

(c) Find the value of c for which the average value of f over the interval $-5 \le x \le c$ is $\frac{1}{2}$.

$$\frac{1}{c--5}\int_{-5}^{c} f(x)\,dx = \frac{1}{2}$$

$$\frac{1}{c+5}\left[-\pi - 1 + 2 + c\right] = \frac{1}{2}$$

$$\frac{-\pi + 1 + c}{c+5} = \frac{1}{2}$$

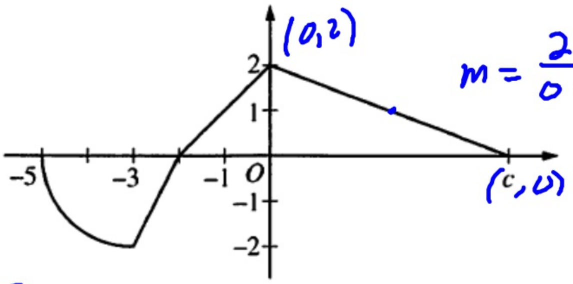
$$c+5 = -2\pi + 2 + 2c$$

$$c = 2\pi + 3$$

$f(g(x))$

$f'(g(x))\,g'(x)$

CHAIN RULE



Graph of f

$m = \dfrac{2-0}{0-c}$

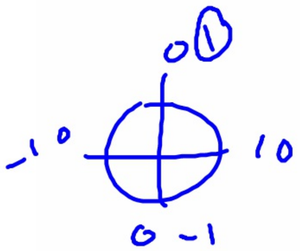
(d) Assume $c > 3$. The function h is defined by $h(x) = f\left(\frac{x}{2}\right)$. Find $h'(6)$ in terms of c.

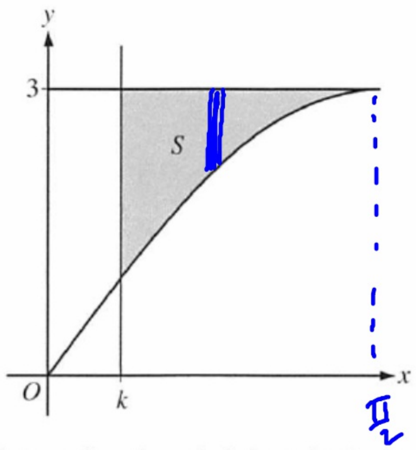
$$h'(x) = f'\left(\tfrac{1}{2}x\right)\left(\tfrac{1}{2}\right)$$

$$h'(6) = f'(3)\,\tfrac{1}{2}$$

$$= \frac{2}{-c}\,\frac{1}{2}$$

$$= -\frac{1}{c}$$





4. Let S be the shaded region in the first quadrant bounded above by the horizontal line $y = 3$, below by the graph of $y = 3\sin x$, and on the left by the vertical line $x = k$, where $0 < k < \frac{\pi}{2}$, as shown in the figure above.

(a) Find the area of S when $k = \frac{\pi}{3}$.

1 sect

$3\sin x = 3$

$\sin x = 1$

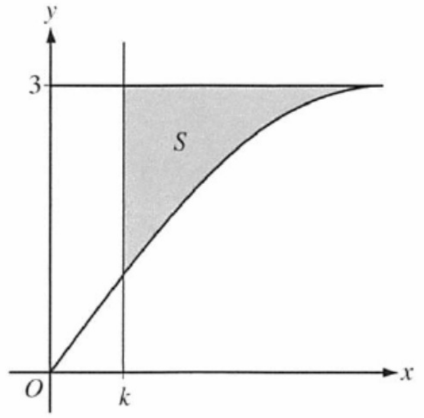
$x = \frac{\pi}{2}$

$$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} [3 - 3\sin x]\,dx = \left[3x + 3\cos x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \left(\frac{3\pi}{2} + 3\cos\frac{\pi}{2}\right) - \left(\pi + 3\cos\frac{\pi}{3}\right)$$

$$= \frac{\pi}{2} - \frac{3}{2}$$

$$\frac{d}{dx}\int_a^{x} f(t)\,dt = f(x)$$



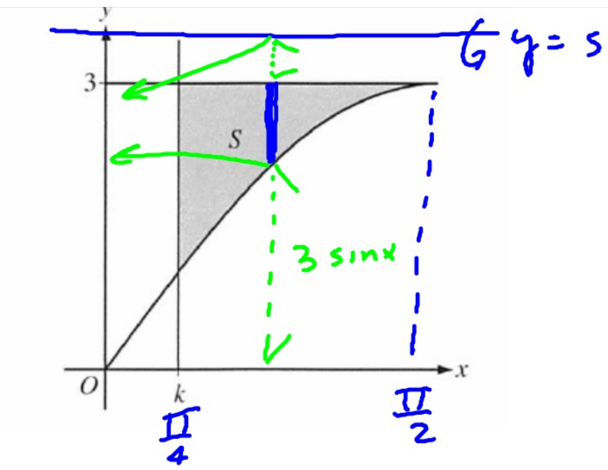
(b) The area of S is a function of k. Find the rate of change of the area of S with respect to k when $k = \frac{\pi}{6}$.

$$S(k) = \int_k^{\frac{\pi}{2}} [3 - 3\sin x]\,dx = -\int_{\frac{\pi}{2}}^{k} [3 - 3\sin x]\,dx$$

$$S'(k) = -(3 - 3\sin k)$$

$$S'\left(\frac{\pi}{6}\right) = -\left(3 - 3\sin\frac{\pi}{6}\right) = -\left(3 - 3\left(\frac{1}{2}\right)\right) = \frac{3}{2} - 3$$

$$= -\frac{3}{2}$$



(c) Region S is revolved about the horizontal line $y = 5$ to form a solid. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid when $k = \frac{\pi}{4}$.

$$V = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[(5 - 3\sin x)^2 - (5 - 3)^2 \right] dx$$

5. For $0 \le t \le 24$ hours, the temperature inside a refrigerator in a kitchen is given by the function W that satisfies the differential equation $\frac{dW}{dt} = \frac{3\cos t}{2W}$. $W(t)$ is measured in degrees Celsius (°C), and t is measured in hours. At time $t = 0$ hours, the temperature inside the refrigerator is 3°C.

(a) Write an equation for the line tangent to the graph of $y = W(t)$ at the point where $\boxed{t = 0}$. Use the equation to approximate the temperature inside the refrigerator at $t = 0.4$ hour.

$(0, 3)$

(x, y)
(t, W)

$$\left.\frac{dW}{dt}\right|_{(0,3)} = \frac{3\cos 0}{6} = \frac{1}{2}$$

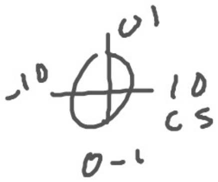
$$W = 3 + \frac{1}{2}t$$

$$W(.4) = 3 + \frac{1}{2}(.4) = 3.2$$

$$\therefore 3 + \frac{1}{2}(.4)\ ^\circ C$$

$$3.2^\circ C$$

(b) Find $y = W(t)$, the particular solution to the differential equation with initial condition $W(0) = 3$.



$$\frac{dW}{dt} = \frac{3\cos t}{2W}$$

$$2W\,dW = 3\cos t\,dt$$

$$W^2 = 3\sin t + C$$

$$W(0) = 3$$

$$9 = 3\sin 0 + C$$

$$9 = C$$

$$W^2 = 3\sin t + 9$$

$$W = \pm\sqrt{3\sin t + 9}$$

$W(0) = 3$

$\therefore W(t) = \sqrt{3\sin t + 9}$

(c) The temperature in the kitchen remains constant at $20°C$ for $0 \le t \le 24$. The cost of operating the refrigerator accumulates at the rate of \$0.001 per hour for each degree that the temperature in the kitchen exceeds the temperature inside the refrigerator. Write, but do not evaluate, an expression involving an integral that can be used to find the cost of operating the refrigerator for the 24-hour interval.

$$.001 \int_0^{24} [20 - W]\, dw$$

$10 - 2 - 1$

$$f(x) = \begin{cases} 10 - 2x - x^2 & \text{for } x \le 1 \\ 3 + 4e^{x-1} & \text{for } x > 1 \end{cases}$$

6. Let f be the function defined above.

   (a) Is f continuous at $x = 1$? Why or why not?

$f(1) = 7$

$$\left.\begin{aligned} \lim_{x \to 1^+} f(x) &= 7 \\ \lim_{x \to 1^-} f(x) &= 7 \end{aligned}\right\} \therefore \lim_{x \to 1} f(x) = 7$$

f is continuous at $x = 1$ because $f(1) = \lim_{x \to 1} f(x)$.

$$f(x) = \begin{cases} 10 - 2x - x^2 & \text{for } x \le 1 \\ 3 + 4e^{x-1} & \text{for } x > 1 \end{cases}$$

(b) Find the absolute minimum value and the absolute maximum value of f on the closed interval $-2 \le x \le 2$. Show the analysis that leads to your conclusion.

$$f'(x) = \begin{cases} -2 - 2x & x < 1 \\ 4e^{x-1} & x > 1 \end{cases}$$

$-2 - 2x = 0 \rightarrow x = -1$

$4e^{x-1} \neq 0$

$f_+'(1) = 4 \quad f_-'(1) = -4$

$f'(1) \nexists$

$f(-2) = 10$

$f(2) = 3 + 4e$

$f(-1) = 11$

$f(1) = 7$

Since f is cont on $[-2, 2]$ by EVT the absolute max is $3 + 4e$ and absolute minimum is 7.

$$f(x) = \begin{cases} 10 - 2x - x^2 & \text{for } x \le 1 \\ 3 + 4e^{x-1} & \text{for } x > 1 \end{cases}$$

(c) Find the value of $\int_0^2 f(x)\,dx$.

$$\int_0^2 f(x)\,dx = \int_0^1 \left[10 - 2x - x^2\right] dx + \int_1^2 \left[3 + 4e^{x-1}\right] dx$$

$$= \left[10x - x^2 - \frac{1}{3}x^3\right]_0^1 + \left[3x + 4e^{x-1}\right]_1^2$$

$$= \left(10 - 1 - \frac{1}{3}\right) - (0) + (6 + 4e) - (3 + 4)$$

$$= \frac{23}{3} + 4e$$