1. If $y = \cos 2x$, then $\dfrac{dy}{dx} =$

(A) $-2\sin 2x$ (B) $-\sin 2x$ (C) $\sin 2x$ (D) $2\sin 2x$ (E) $2\sin x$

$-2\sin 2x$

2. $\int x^2 \left(x^3 - 1 \right)^{10} dx =$

(A) $\frac{x^3}{3} \left(\frac{x^4}{4} - x \right)^{10} + C$

(B) $\frac{\left(x^3 - 1 \right)^{11}}{11} + C$

(C) $\frac{x^2 \left(x^3 - 1 \right)^{11}}{11} + C$

(D) $\frac{\left(x^3 - 1 \right)^{11}}{33} + C$

(E) $\frac{x^3 \left(x^3 - 1 \right)^{11}}{33} + C$

$u = x^3 - 1$

$\frac{1}{3} du = x^2 dx$

$\frac{1}{3} \int u^{10} \, du = \frac{1}{3} \cdot \frac{1}{11} \left(x^3 - 1 \right)^{11} + C$

3. $\lim\limits_{x \to \infty} \dfrac{\sqrt{9x^4 + 1}}{4x^2 + 3}$ is

(A) $\dfrac{1}{3}$ (B) $\dfrac{3}{4}$ (C) $\dfrac{3}{2}$ (D) $\dfrac{9}{4}$ (E) infinite

$$\frac{\sqrt{9}\ \sqrt{x^4}^{\,x^2}}{4x^2} \qquad \frac{3}{4}$$

4. If $y = \left(\dfrac{x}{x+1}\right)^5$, then $\dfrac{dy}{dx} =$

(A) $5(1+x)^4$ (B) $\dfrac{x^4}{(x+1)^4}$ (C) $\dfrac{5x^4}{(x+1)^4}$ (D) $\dfrac{5x^4}{(x+1)^6}$ (E) $\dfrac{5x^4(2x+1)}{(x+1)^6}$

$$\frac{dy}{dx} = 5\left(\frac{x}{x+1}\right)^4 \left(\frac{(x+1)-x}{(x+1)^2}\right)$$

$$= 5\left(\frac{x}{x+1}\right)^4 \left(\frac{1}{(x+1)^2}\right)$$

$$= \frac{5x^4}{(x+1)^2}$$

t (minutes)	0	4	7	9
$r(t)$ (gallons per minute)	9	6	4	3

5. Water is flowing into a tank at the rate $r(t)$, where $r(t)$ is measured in gallons per minute and t is measured in minutes. The tank contains 15 gallons of water at time $t = 0$. Values of $r(t)$ for selected values of t are given in the table above. Using a trapezoidal sum with the three intervals indicated by the table, what is the approximation of the number of gallons of water in the tank at time $t = 9$?

(A) 52   (B) 57   (C) 67   (D) 77   (E) 79

$$15 + \int_0^9 r(t)\,dt$$
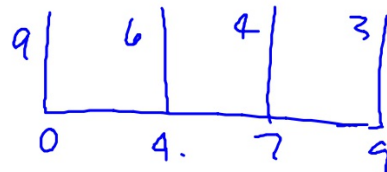
$$15 + \left[\tfrac{1}{2}(15)(4) + \tfrac{1}{2}(10)(3) + \tfrac{1}{2}(7)(2) \right]$$

$$15 + 30 + 15 + 7$$

$$67$$

6. The slope of the line tangent to the graph of $y = \ln(1 - x)$ at $x = -1$ is

(A) -1 (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) $\ln 2$ (E) 1

$$\frac{dy}{dx} = \frac{-1}{1-x}$$

$$\left.\frac{dy}{dx}\right|_{x=-1} = \frac{-1}{1--1} = -\frac{1}{2}$$

$$D_x\left[\ln \bigstar\right] = \frac{\bigstar'}{\bigstar}$$

7. For which of the following pairs of functions f and g is $\displaystyle\lim_{x\to\infty} \frac{f(x)}{g(x)}$ infinite?

(A) $f(x) = x^2 + 2x$ and $g(x) = x^2 + \ln x$

(B) $f(x) = 3x^3$ and $g(x) = x^4$

(C) $f(x) = 3^x$ and $g(x) = x^3$

(D) $f(x) = 3e^x + x^3$ and $g(x) = 2e^x + x^2$

(E) $f(x) = \ln(3x)$ and $g(x) = \ln(2x)$

$$\frac{x^2+2x}{x^2+\ln x}$$

$$\frac{3x^3}{x^4}$$

$$\frac{\boxed{3^x}}{\boxed{x^3}} \quad \infty$$

$$\frac{2^x}{x^{1000}}$$

$$\frac{N}{D}$$

8. $\displaystyle\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx =$

(A) -2 (B) $-\dfrac{2}{15}$ (C) 1 (D) 2 (E) 5

$$\frac{u^{1/2}}{\frac{1}{2}}$$

$u = x^2 + 9$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$x = 0 \rightarrow u = 9$

$x = 4 \rightarrow u = 25$

$$\frac{1}{2}\int_9^{25} u^{-1/2}\,du = \frac{1}{2}\cdot 2u^{1/2}\Big|_9^{25}$$

$$= \sqrt{25} - \sqrt{9}$$

$$= 5 - 3$$

$$= 2$$

9. Let f be the function with derivative given by $f'(x) = \dfrac{-2x}{\left(1 + x^2\right)^2}$. On what interval is f decreasing?

(A) $[0, \infty)$ only

(B) $(-\infty, 0]$ only

(C) $\left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$ only

(D) $(-\infty, \infty)$

(E) There is no such interval.

$f'(x) < 0$

$-2x < 0$

$x > 0$