

AP CALCULUS  
THE FUNDAMENTAL THEOREMS B

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$$1. \frac{d}{dx} \int_0^x \sqrt{4+t^6} dt = (\sqrt{4+x^6})(1) - \sqrt{4}(0)$$

$$= \sqrt{4+x^6}$$

$$2. \frac{d}{dx} \int_2^x \frac{1}{t^4+4} dt = \frac{1}{x^4+4}(1) - \frac{1}{2^4+4}(0)$$

$$= \frac{1}{x^4+4}$$

$$3. \frac{d}{dx} \int_x^3 \sqrt{\sin t} dt = \frac{d}{dx} \left[ - \int_3^x \sqrt{\sin t} dt \right]$$

$$= -\sqrt{\sin x}(1) - \sqrt{\sin 3}(0)$$

$$= -\sqrt{\sin x}$$

$$4. \frac{d}{dx} \int_{-x}^x \frac{1}{3+t^2} dt = \frac{1}{3+x^2}(1) - \frac{1}{3+(-x)^2}(-1)$$

$$= \frac{2}{3+x^2}$$

$$5. \frac{d}{dx} \int_1^{x^3} \sqrt[3]{t^2+1} dt = \sqrt[3]{x^6+1}(3x^2) - \sqrt[3]{1+1}(0)$$

$$= 3x^2 \sqrt[3]{x^6+1}$$

$$6. \frac{d}{dx} \int_2^{\tan x} \frac{1}{1+t^2} dt = \frac{1}{1+\tan^2 x}(\sec^2 x) - \frac{1}{1+2^2}(0)$$

$$= \frac{\sec^2 x}{1+\tan^2 x}$$

$$= \frac{\sec^2 x}{\sec^2 x}$$

$$= 1$$

$$7. F(2) = \int_2^2 \sqrt{3t^2+1} dt = 0$$

$$F'(x) = \sqrt{3x^2+1} \rightarrow F'(2) = \sqrt{13}$$

$$F''(x) = \frac{3x}{\sqrt{3x^2+1}} \rightarrow F''(2) = \frac{6}{\sqrt{13}}$$

$$8. F(0) = \int_0^0 \frac{\cos t}{t^2+3} dt = 0$$

$$F'(x) = \frac{\cos x}{x^2+3} \rightarrow F'(0) = \frac{1}{3}$$

$$F''(x) = \frac{(x^2+3)(-\sin x) - (\cos x)(2x)}{(x^2+3)^2} \rightarrow F''(0) = 0$$

$$9. \frac{d}{dx} \int_{2x}^{3x} \frac{u-1}{u+1} du = \frac{3x-1}{3x+1} (3) - \frac{2x-1}{2x+1} (2)$$

$$10. \frac{d}{dx} \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}} dt = \frac{1}{\sqrt{2+x^8}} (2x) - \frac{1}{\sqrt{2+\tan^4 x}} (\sec^2 x)$$

$$11. \frac{d}{dx} \int_{\sqrt{x}}^{x^3} \sqrt{t} dt = 3x^2 \sqrt{x^3} - \frac{\sqrt[4]{x}}{2\sqrt{x}}$$

$$12. \frac{d}{dx} \int_{\cos x}^{5x} \cos u^2 du = 5 \cos 25x^2 - [\cos(\cos^2 x)](-\sin x)$$

$$13. F(x) = -3 + \int_5^x (t-3)^3 dt$$

$$F(5) = -3 + 0 = -3$$

$$F'(x) = 0 + (x-3)^3 \rightarrow F'(5) = 8$$

$$F''(x) = 3(x-3)^2 \rightarrow F''(5) = 12$$

$$14. G(3) = 0$$

$$G'(x) = \ln x$$

$$G'(3) = \ln 3$$

$$\therefore G(3) - G'(3) = -\ln 3$$

$$15. F(x) = -7 + \int_8^x \sqrt{2t^2 - 5} dt$$

$$F(8) = -7 + 0 = -7$$

$$F'(x) = \sqrt{2x^2 - 5} \rightarrow F'(8) = \sqrt{123}$$

$$F''(x) = \frac{2x}{\sqrt{2x^2 - 5}} \rightarrow F''(8) = \frac{16}{\sqrt{123}}$$