1. $\int_0^1 (2x+1)\,\mathrm{d}x = (x^2 - x)\Big|_0^1 = (1-1) - (0) = 0$

2. $$\begin{aligned}\int_0^1 (x^4+x)(4x^3+1)\,\mathrm{d}x &= \int_0^1 (4x^7 + 5x^4 + x)\,\mathrm{d}x \\ &= \left(\frac{1}{2}x^8 + x^5 + \frac{1}{2}x^2\right)\Big|_0^1 \\ &= \left(\frac{1}{2} + 1 + \frac{1}{2}\right) - 0 \\ &= 2\end{aligned}$$

3. $u = x - 1 \longrightarrow du = dx$
   $x = u + 1$
   $x = 1 \longrightarrow u = 0$
   $x = 2 \longrightarrow u = 1$

$$\begin{aligned}\int_1^2 x\sqrt{x-1}\,\mathrm{d}x &= \int_0^1 u^{1/2}(u+1)\,\mathrm{d}u \\ &= \int_0^1 \left(u^{3/2} + u^{1/2}\right)\mathrm{d}u \\ &= \left(\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right)\Big|_0^1 \\ &= \left(\frac{2}{5} + \frac{2}{3}\right) - 0 \\ &= \frac{16}{15}\end{aligned}$$

4. $$\begin{aligned}\int_0^1 \cos\pi x\,\mathrm{d}x &= \frac{1}{\pi}\sin\pi x\Big|_0^1 \\ &= \left(\frac{1}{\pi}\sin\pi\right) - \left(\frac{1}{\pi}\sin 0\right) \\ &= 0\end{aligned}$$

5. $u = 1 + \frac{1}{x} \longrightarrow du = -\frac{1}{x^2}\,dx$

$x = 1 \longrightarrow u = 2$

$x = 4 \longrightarrow u = \frac{5}{4}$

$$\begin{aligned} \int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,\mathrm{d}x &= -\int_2^{5/4} u^{1/2}\,\mathrm{d}u \\ &= \int_{5/4}^2 u^{1/2}\,\mathrm{d}u \\ &= \frac{2}{3}u^{3/2}\Big|_{5/4}^{2} \\ &= \frac{4\sqrt{2}}{3} - \frac{5\sqrt{5}}{12} \end{aligned}$$

6. $$\begin{aligned} \int_0^{\pi/3} \frac{\sin x}{\cos^2 x}\,\mathrm{d}x &= \int_0^{\pi/3} \tan x \sec x\,\mathrm{d}x \\ &= \sec x\Big|_0^{\pi/3} \\ &= 1 \end{aligned}$$

7. $u = 1 + 2x \longrightarrow \frac{1}{2}du = dx$

$x = 0 \longrightarrow u = 1$

$x = 13 \longrightarrow u = 27$

$$\begin{aligned} \int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}\,\mathrm{d}x &= \frac{1}{2}\int_1^{27} u^{-2/3}\,\mathrm{d}u \\ &= \frac{3}{2}u^{1/3}\Big|_1^{27} \\ &= \frac{9}{2} - \frac{3}{2} \\ &= 3 \end{aligned}$$

8. $u = x - 2 \longrightarrow du = dx$

$x = 0 \longrightarrow u = -2$

$x = 4 \longrightarrow u = 2$

$$\begin{aligned}
\int_0^4 \frac{dx}{(x-2)^3}\,\mathrm{d}x &= \int_{-2}^{2} u^{-3}\,\mathrm{d}u \\
&= -\frac{1}{2}u^{-2}\Big|_{-2}^{2} \\
&= -\frac{1}{2u^2}\Big|_{-2}^{2} \\
&= \left(-\frac{1}{8}\right) - \left(-\frac{1}{8}\right) \\
&= 0
\end{aligned}$$

9. $u = x^2 + a^2 \longrightarrow \frac{1}{2}du = x\ dx$

$x = 0 \longrightarrow u = a^2$

$x = a \longrightarrow u = 2a^2$

$$\begin{aligned}
\int_0^a x\sqrt{x^2+a^2}\,\mathrm{d}x &= \frac{1}{2}\int_{a^2}^{2a^2} u^{1/2}\,\mathrm{d}u \\
&= \frac{1}{2}\frac{2}{3}u^{3/2}\Big|_{a^2}^{2a^2} \\
&= \frac{1}{3}\sqrt{8a^6} - \frac{1}{3}\sqrt{a^6}
\end{aligned}$$

10. It is critical that you are able to separate absolute valued integrals but whenever we have the absolute value of a linear expression, the preferred method is to draw the graph and add up the triangles.

Solution by separating the integral:

$$\begin{aligned}
\int_{-2}^{5} |x-3|\,\mathrm{d}x &= \int_{-2}^{3} (3-x)\,\mathrm{d}x + \int_{3}^{5} (x-3)\,\mathrm{d}x \\
&= \left(3x - \frac{1}{2}x^2\right)\Big|_{-2}^{3} + \left(\frac{1}{2}x^2 - 3x\right)\Big|_{3}^{5} \\
&= \left(9 - \frac{9}{2}\right) - (-6-2) + \left(\frac{25}{2} - 15\right) - \left(\frac{9}{2} - 9\right) \\
&= \frac{29}{2}
\end{aligned}$$

Solution via triangles: $\displaystyle\int_{-2}^{5} |x-3|\,\mathrm{d}x = \frac{1}{2}(5)(5) + \frac{1}{2}(2)(2)$

$$= \frac{29}{2}$$

11. First, divide the numerator by the denominator using polynomial long division or synthetic division.

$$\begin{aligned}
\int_0^1 \frac{x^3-1}{x+1}\,\mathrm{d}x &= \int_0^1 \left(x^2 - x + 1 - \frac{2}{x+1}\right)\mathrm{d}x \\
&= \left[\frac{1}{3}x^2 - \frac{1}{2}x^2 + x - 2\ln|x+1|\right]\Big|_0^1 \\
&= \left(\frac{1}{3} - \frac{1}{2} + 1 - 2\ln 2\right) - (0 - 0 + 0 - 2\ln 1) \\
&= \frac{5}{6} - 2\ln 2
\end{aligned}$$

12. $$\begin{aligned}
\int_0^{\pi/2} \sin 2x\,\mathrm{d}x &= -\frac{1}{2}\cos 2x\Big|_0^{\pi/2} \\
&= \left(-\frac{1}{2}\cos\pi\right) - \left(-\frac{1}{2}\cos 0\right) \\
&= 1
\end{aligned}$$

13. $u = 3x^2 - 1 \longrightarrow \frac{1}{6}\,du = x\,dx$

$x = 1 \longrightarrow u = 2$
$x = 3 \longrightarrow u = 26$

$$\begin{aligned}
\int_1^3 \frac{x}{(3x^2-1)^3}\,\mathrm{d}x &= \frac{1}{6}\int_2^{26} u^{-3}\,\mathrm{d}u \\
&= \frac{1}{6}\frac{u^{-2}}{-2}\Big|_2^{26} \\
&= -\frac{1}{12u^2}\Big|_2^{26} \\
&= \frac{7}{338}
\end{aligned}$$

14. $u = x + 3 \longrightarrow du = dx$
$x = u - 3 \longrightarrow x + 2 = u - 1$
$x = -2 \longrightarrow u = 1$
$x = 1 \longrightarrow u = 4$

$$\begin{aligned}
\int_{-2}^1 (x+2)\sqrt{x+3}\,\mathrm{d}x &= \int_1^4 u^{1/2}(u-1)\,\mathrm{d}u \\
&= \int_1^4 \left(u^{3/2} - u^{1/2}\right)\mathrm{d}u \\
&= \left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right)\Big|_1^4 \\
&= \left[\frac{2}{5}(32) - \frac{2}{3}(8)\right] - \left[\frac{2}{5} - \frac{2}{3}\right] \\
&= \frac{116}{15}
\end{aligned}$$

15.
$$\begin{aligned}\int_2^4 \frac{w^4 - w}{w^3}\,\mathrm{d}w &= \int_2^4 (w - w^{-2})\,\mathrm{d}w \\ &= \left(\frac{1}{2}w^2 + w^{-1}\right)\Big|_2^4 \\ &= \left(8 + \frac{1}{4}\right) - \left(2 + \frac{1}{2}\right) \\ &= \frac{23}{4}\end{aligned}$$