

1.  $\Delta x = \frac{2}{n}$  and  $x_i = i\Delta x$

$$\begin{aligned}
 A &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n i^2 (\Delta x)^2 \Delta x \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{8}{n^3} i^2 \\
 &= \lim_{n \rightarrow \infty} \left[ \frac{8}{n^3} \frac{2n^3 + 3n^2 + n}{6} \right] \\
 &= \left( \frac{8}{6} \right) 2 \\
 &= \frac{8}{3}
 \end{aligned}$$

2.  $\Delta x = \frac{2}{n}$  and  $x_i = 1 + i\Delta x$

$$\begin{aligned}
 A &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n (1 + i\Delta x)^2 \Delta x \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n (1 + 2i\Delta x + i^2(\Delta x)^2) \Delta x \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n (\Delta x + 2i(\Delta x)^2 + i^2(\Delta x)^3) \\
 &= \lim_{n \rightarrow \infty} \left[ 2 + \frac{8}{n^2} \frac{n^2 + n}{2} + \frac{8}{n^3} \frac{2n^3 + 3n^2 + n}{6} \right] \\
 &= \frac{26}{3}
 \end{aligned}$$

$$3. \Delta x = \frac{3}{n} \text{ and } x_i = 1 + i\Delta x$$

$$\begin{aligned} A &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [2(1 + i\Delta x)] \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [2\Delta x + 2i(\Delta x)^2] \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left( \frac{6}{n} + \frac{18}{n^2} i \right) \\ &= \lim_{n \rightarrow \infty} \left[ 6 + \frac{18}{n^2} \frac{n^2 + n}{2} \right] \\ &= 15 \end{aligned}$$

$$4. \Delta x = \frac{1}{n} \text{ and } x_i = 1 + i\Delta x$$

$$\begin{aligned} A &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [4 - (1 + i\Delta x)^2] \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [4 - (1 + 2i\Delta x + i^2(\Delta x)^2)] \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [3\Delta x - 2i(\Delta x)^2 - i^2(\Delta x)^3] \\ &= \lim_{n \rightarrow \infty} \left[ 3 - \frac{2}{n^2} \frac{n^2 + n}{2} - \frac{1}{n^3} \frac{2n^3 + 3n^2 + n}{6} \right] \\ &= \frac{5}{3} \end{aligned}$$

$$5. \Delta x = \frac{3}{n} \text{ and } x_i = 1 + i\Delta x$$

$$\begin{aligned} A &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [(1 + i\Delta x)^2 + 3(1 + i\Delta x) - 2] \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [1 + 2i\Delta x + i^2(\Delta x)^2 + 3 + 3i\Delta x - 2] \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [2\Delta x + 5i(\Delta x)^2 + i^2(\Delta x)^3] \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \frac{6}{n} + \frac{45}{n^2} i + \frac{27}{n^3} i^2 \right] \\ &= \lim_{n \rightarrow \infty} \left[ \frac{6}{n} + \frac{45}{n^2} \frac{n^2 + n}{2} + \frac{27}{n^3} \frac{2n^3 + 3n^2 + n}{6} \right] \\ &= \frac{75}{2} \end{aligned}$$

$$6. \Delta x = \frac{5}{n} \text{ and } x_i = -3 + i\Delta x$$

$$\begin{aligned} A &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [2(i\Delta x - 3)^2 - 4(i\Delta x - 3) + 5] \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [2(i^2(\Delta x)^2 - 6i\Delta x + 9) - 4i\Delta x + 12 + 5] \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [2(i^2(\Delta x)^2 - 12i\Delta x + 18 - 4i\Delta x + 17)] \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n [2(i^2(\Delta x)^3 - 16i(\Delta x)^2 + 35\Delta x)] \\ &= \lim_{n \rightarrow \infty} \left[ \frac{250}{n^3} \frac{2n^3 + 3n^2 + n}{6} - \frac{400}{n^2} \frac{n^2 + n}{2} + 175 \right] \\ &= \frac{175}{3} \end{aligned}$$