1. $\Delta x = \dfrac{2}{n}$ and $c_i = x_i = 1 + i\Delta x$

$$\begin{aligned}
\int_1^3 (1+2x)\,\mathrm{d}x &= \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[1 + 2(1 + i\Delta x)\right]\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[3\Delta x + 2i(\Delta x)^2\right] \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{6}{n} + \frac{8}{n^2}\,i\right] \\
&= \lim_{n\to\infty} \left[6 + \frac{8}{n^2}\,\frac{n^2+n}{2}\right] \\
&= 10
\end{aligned}$$

2. $\Delta x = \dfrac{2}{n}$ and $c_i = x_i = 1 + i\Delta x$

$$\begin{aligned}
\int_1^3 (x^2 + 4x)\,\mathrm{d}x &= \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[(1 + i\Delta x)^2 + 4(1 + i\Delta x)\right] \Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[1 + 2i\Delta x + i^2(\Delta x)^2 + 4 + 4i\Delta x\right] \Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[5\Delta x + 6i(\Delta x)^2 + i^2(\Delta x)^3\right] \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{10}{n} + \frac{24}{n^2}\, i + \frac{8}{n^3}\, i^2\right] \\
&= \lim_{n\to\infty} \left[10 + \frac{24}{n^2}\,\frac{n^2 + n}{2} + \frac{8}{n^3}\,\frac{2n^3 + 3n^2 + n}{6}\right] \\
&= \frac{74}{3}
\end{aligned}$$

3. $\Delta x = \dfrac{4}{n}$ and $c_i = x_i = i\Delta x$

$$\begin{aligned}
\int_0^4 (x^2 + 2)\,\mathrm{d}x &= \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[i^2(\Delta x)^2 + 2\right] \Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[i^2(\Delta x)^3 + 2\Delta x\right] \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{64}{n^3} i^2 + \frac{8}{n}\right] \\
&= \lim_{n\to\infty} \left[\frac{64}{n^3}\,\frac{2n^3 + 3n^2 + n}{6} + 8\right] \\
&= \frac{88}{3}
\end{aligned}$$

4. Try to separate the argument into two parts ... one part which will be Δx and the other part $i\Delta x$.

   Try letting $\Delta x = \dfrac{1}{n}$ which leaves $\dfrac{i^4}{n^4}$ for $i\Delta x$.

   Now, what function would you have to have in order for $f(i\Delta x)$ to be $\dfrac{i^4}{n^4}$?

   Clearly $f(x) = x^4$ and $a = 0$ and $b = 1$ $\therefore$ $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^4}{n^5} = \int_0^1 x^4\,\mathrm{d}x$.

5. This one is pretty clear ... $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\sin\left(1+\frac{3i}{n}\right)\frac{3}{n} = \int_1^4 \sin(x)\,\mathrm{d}x$

6. $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left[3\left(2+\frac{2i}{n}\right)-6\right]\frac{2}{n} = \int_1^3 (3x^5-6)\,\mathrm{d}x$

7. $\displaystyle\int_1^{12} f(x)\,\mathrm{d}x$

8. $\displaystyle\int_0^{8} f(x)\,\mathrm{d}x$

9. $\displaystyle\int_7^{10} f(x)\,\mathrm{d}x$

10. $\displaystyle\int_0^{6} f(x)\,\mathrm{d}x$

11. $\displaystyle\int_4^{6} h(x)\,\mathrm{d}x = 12$

    $\displaystyle\int_4^6 [5h(x)-7]\,\mathrm{d}x = 5\int_4^6 h(x)\,\mathrm{d}x - \int_4^6 7\,\mathrm{d}x = 5(12)-7(6-4) = 46$

12. $\displaystyle\int_1^{10} p(x)\,\mathrm{d}x = 24$

    $\displaystyle\int_1^{10} [3p(x)-2]\,\mathrm{d}x = 3\int_1^{10} p(x)\,\mathrm{d}x - \int_1^{10} 2\,\mathrm{d}x = 3(24)-2(10-1) = 54$

13. $\displaystyle\int_2^5 [4f(x)-3]\,\mathrm{d}x = 4\int_2^5 f(x)\,\mathrm{d}x - \int_2^5 3\,\mathrm{d}x = 4(3a-2b)-3(5-2) = 12a-8b-9$

14. $\displaystyle\int_a^b [6g(x)+6]\,\mathrm{d}x = 6\int_a^b g(x)\,\mathrm{d}x + \int_a^b 6\,\mathrm{d}x = 6(3a+11b)+6(b-a) = 12a+72b$

15. $\displaystyle\int_a^b [3r(x)-7]\,\mathrm{d}x = 3\int_a^b r(x)\,\mathrm{d}x - \int_a^b 7\,\mathrm{d}x = 3(8a+9b)-7(b-a) = 31a+20b$

16. $\displaystyle\int_a^b [9q(x)-1]\,\mathrm{d}x = 9\int_a^b q(x)\,\mathrm{d}x - \int_a^b 1\,\mathrm{d}x = 9(a-9b)-1(b-a) = 10a-82b$

17. $\displaystyle\int_5^7 f(x)\,\mathrm{d}x = 20-6a+2b$

    $\displaystyle\int_5^7 [4f(x)-2]\,\mathrm{d}x = 4\int_5^7 f(x)\,\mathrm{d}x - \int_5^7 2\,\mathrm{d}x = 4(20-6a+2b)-2(7-5) = -24a+8b+76$

18. $\displaystyle\int_1^9 h(x)\,\mathrm{d}x = 4a-2b+51$

    $\displaystyle\int_1^9 [8h(x)-5]\,\mathrm{d}x = 8\int_1^9 h(x)\,\mathrm{d}x - \int_1^9 5\,\mathrm{d}x = 8(4a-2b+51)-5(9-1) = 32a-16b+368$

19. $\int_1^4 r(x)\,\mathrm{d}x = 9$

20. $\int_2^6 t(x)\,\mathrm{d}x = 13$