

AP CALCULUS
SIGMA NOTATION

1. $\Delta x = \frac{2}{5}$
 $x_0 = 0$
 $x_1 = \frac{2}{5}$
 $x_2 = \frac{4}{5}$
 $x_3 = \frac{6}{5}$
 $x_4 = \frac{8}{5}$
 $x_5 = 2$ (Not used on left sum.)

$$A \approx f(0)\frac{2}{5} + f\left(\frac{2}{5}\right)\frac{2}{5} + \dots + f\left(\frac{8}{5}\right)\frac{2}{5} \approx \frac{252}{25}$$

2. $\Delta x = 1$
 $x_0 = -3$ (Not used on right sum.)
 $x_1 = -2$
 $x_2 = -1$
 $x_3 = 0$
 $x_4 = 1$
 $x_5 = 2$

$$A \approx f(-2)(1) + f(-1)(1) + \dots + f(2)(1) \approx 20$$

3. $\Delta x = 1$
 $x_0 = 1$
 $x_1 = 2$
 $x_2 = 3$
 $x_3 = 4$
 $x_4 = 5$
 $x_5 = 6$

$$\begin{aligned}m_1 &= \frac{3}{2} \\m_2 &= \frac{5}{2} \\m_3 &= \frac{7}{2} \\m_4 &= \frac{9}{2} \\m_5 &= \frac{11}{2}\end{aligned}$$

$$A \approx f\left(\frac{3}{2}\right)(1) + f\left(\frac{5}{2}\right)(1) + \dots + f\left(\frac{11}{2}\right)(1) \approx \frac{405}{4}$$

4. $\Delta x = \frac{3}{4}$
 $x_0 = 0$
 $x_1 = \frac{3}{4}$
 $x_2 = \frac{3}{2}$
 $x_3 = \frac{9}{4}$
 $x_4 = 3$ (Not used on left sum.)

$$A \approx f(0)\frac{3}{4} + f\left(\frac{3}{4}\right)\frac{3}{4} + f\left(\frac{3}{2}\right)\frac{3}{4} + f\left(\frac{9}{4}\right)\frac{3}{4} \approx \frac{369}{32}$$

5. $\Delta x = \frac{3}{4}$
 $x_0 = 0$ (Not used on right sum.)
 $x_1 = \frac{3}{4}$
 $x_2 = \frac{3}{2}$
 $x_3 = \frac{9}{4}$
 $x_4 = 3$

$$A \approx f\left(\frac{3}{4}\right)\frac{3}{4} + f\left(\frac{3}{2}\right)\frac{3}{4} + f\left(\frac{9}{4}\right)\frac{3}{4} + f(3)\frac{3}{4} \approx \frac{513}{32}$$

6. $\Delta x = \frac{3}{4}$
 $x_0 = 0$
 $x_1 = \frac{3}{4}$
 $x_2 = \frac{3}{2}$
 $x_3 = \frac{9}{4}$
 $x_4 = 3$

$$\begin{aligned}m_1 &= \frac{3}{8} \\m_2 &= \frac{9}{8} \\m_3 &= \frac{15}{8} \\m_4 &= \frac{21}{8}\end{aligned}$$

$$A \approx \frac{3}{4} \left[f\left(\frac{3}{8}\right) + f\left(\frac{9}{8}\right) + f\left(\frac{15}{8}\right) + f\left(\frac{21}{8}\right) \right] \approx \frac{855}{64}$$

Note: You can factor out the Δx to make the calculation less cumbersome.

7. $\Delta x = 1$
 $x_0 = -1$
 $x_1 = 0$
 $x_2 = 1$
 $x_3 = 2$
 $x_4 = 3$
 $x_5 = 4$ (Not used on left sum.)

$$A \approx 1 [f(-1) + f(0) + f(1) + f(2) + f(3)] \approx 35$$

8. $\Delta x = 1$
 $x_0 = -1$ (Not used on right sum.)
 $x_1 = 0$
 $x_2 = 1$
 $x_3 = 2$
 $x_4 = 3$
 $x_5 = 4$

$$A \approx 1 [f(0) + f(1) + f(2) + f(3) + f(4)] \approx 40$$

9. $\Delta x = 1$
 $x_0 = -1$
 $x_1 = 0$
 $x_2 = 1$
 $x_3 = 2$
 $x_4 = 3$
 $x_5 = 4$

$$\begin{aligned}m_1 &= -\frac{1}{2} \\m_2 &= \frac{1}{2} \\m_3 &= \frac{3}{2} \\m_4 &= \frac{5}{2} \\m_5 &= \frac{7}{2}\end{aligned}$$

$$A \approx 1 \left[f\left(\frac{-1}{2}\right) \right] + f\left(\frac{1}{2}\right) + f\left(\frac{3}{2}\right) + f\left(\frac{5}{2}\right) + f\left(\frac{7}{2}\right) \approx 35$$