1. $u = 3x + 4 \longrightarrow du = 3\,dx \longrightarrow \dfrac{1}{3}\,du = dx$

$$\begin{aligned}\int \sqrt{3x+4}\,\mathrm{d}x &= \frac{1}{3}\int u^{1/2}\,\mathrm{d}u \\ &= \frac{1}{3}\frac{2}{3}u^{3/2} + C \\ &= \frac{2}{9}u^{3/2} + C \\ &= \frac{2}{9}(3x+4)^{3/2} + C\end{aligned}$$

2. $u = 5 + 2x^3 \longrightarrow du = 6x^2\,dx \longrightarrow \dfrac{1}{6}\,du = x^2\,dx$

$$\begin{aligned}\int x^2(5+2x^3)^5\,\mathrm{d}x &= \frac{1}{6}\int u^5\,\mathrm{d}u \\ &= \frac{1}{6}\frac{1}{6}u^6 + C \\ &= \frac{1}{36}u^6 + C \\ &= \frac{1}{36}(5+2x^3)^6 + C\end{aligned}$$

3. $u = x^2 \longrightarrow du = 2x\,dx \longrightarrow \dfrac{1}{2}\,du = x\,dx$

$$\begin{aligned}\int x\cos x^2\,\mathrm{d}x &= \frac{1}{2}\int \cos u\,\mathrm{d}u \\ &= \frac{1}{2}\sin u + C \\ &= \frac{1}{2}\sin x^2 + C\end{aligned}$$

4. $u = 5 + 2x^3 \longrightarrow du = 6x^2\,dx \longrightarrow \dfrac{1}{6}\,du = x^2\,dx$

$$\begin{aligned}\int x^2(5+2x^3)^8\,\mathrm{d}x &= \frac{1}{6}\int u^8\,\mathrm{d}u \\ &= \frac{1}{6}\frac{1}{9}u^9 + C \\ &= \frac{1}{54}u^9 + C \\ &= \frac{1}{54}(5+2x^3)^9 + C\end{aligned}$$

5. $u = 1 - 8x^3 \longrightarrow du = -24x^2\,dx \longrightarrow -\dfrac{1}{6}\,du = 4x^2\,dx$

$$\begin{aligned}\int \frac{4x^2}{(1-8x^3)^4}\,\mathrm{d}x &= -\frac{1}{6}\int u^{-4}\,\mathrm{d}u \\ &= \left(-\frac{1}{6}\right)\left(-\frac{1}{3}\right)u^{-3} + C \\ &= \frac{1}{18}u^{-3} + C \\ &= \frac{1}{18(1-8x^3)^3} + C\end{aligned}$$

6. $u = 1 + x \longrightarrow du = dx$

$x = u - 1 \longrightarrow x^2 = u^2 - 2u + 1$

$$\begin{aligned}\int x^2\sqrt{1+x}\,\mathrm{d}x &= \int u^{1/2}(u^2 - 2u + 1)\,\mathrm{d}u \\ &= \int \left(u^{5/2} - 2u^{3/2} + u^{1/2}\right)\mathrm{d}u \\ &= \frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C \\ &= \frac{2}{7}(1+x)^{7/2} - \frac{4}{5}(1+x)^{5/2} + \frac{2}{3}(1+x)^{3/2} + C\end{aligned}$$

7. $u = \sqrt{x} \longrightarrow du = \dfrac{1}{2\sqrt{x}}\,dx \longrightarrow 2\,du = \dfrac{1}{\sqrt{x}}\,dx$

$$\begin{aligned}\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,\mathrm{d}x &= 2\int \sin u\,\mathrm{d}u \\ &= -2\cos u + C \\ &= -2\cos\sqrt{x} + C\end{aligned}$$

8. $u = 1 - \cos x \longrightarrow du = \sin x\,dx$

$$\begin{aligned}\int \sin\sqrt{1-\cos x}\,\mathrm{d}x &= \int u^{1/2}\,\mathrm{d}u \\ &= \frac{2}{3}u^{3/2} + C \\ &= \frac{2}{3}(1-\cos x)^{3/2} + C\end{aligned}$$

9. $u = x + 2 \longrightarrow du = dx$
$x = u - 2$

$$\begin{aligned}\int x\sqrt{x+2}\,\mathrm{d}x &= \int u^{1/2}(u-2)\,\mathrm{d}u \\ &= \int \left(u^{3/2} - 2u^{1/2}\right)\mathrm{d}u \\ &= \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C \\ &= \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C\end{aligned}$$

10. $u = t + 3 \longrightarrow du = dt$
$t = u - 3$

$$\begin{aligned}\int \frac{t}{\sqrt{t+3}}\,\mathrm{d}t &= \int u^{-1/2}(u-3)\,\mathrm{d}u \\ &= \int \left(u^{1/2} - 3u^{-1/2}\right)\mathrm{d}u \\ &= \frac{2}{3}u^{3/2} - 6u^{1/2} + C \\ &= \frac{2}{3}(t+3)^{3/2} - 6(t+3)^{1/2} + C\end{aligned}$$

11. $u = 4\beta \longrightarrow du = 4\,d\beta \longrightarrow \frac{1}{4}\,du = d\beta$

$$\begin{aligned}\int \cos\beta\,\mathrm{d}\beta &= \frac{1}{4}\int \cos u\,\mathrm{d}u \\ &= \frac{1}{4}\sin u + C \\ &= \frac{1}{4}\sin 4\beta + C\end{aligned}$$

12. $u = 5x \longrightarrow du = 5\,dx \longrightarrow \frac{1}{5}\,du = dx$

$$\begin{aligned}\int \sec^2 5x\,\mathrm{d}x &= \frac{1}{5}\int \sec^2 u\,\mathrm{d}u \\ &= \frac{1}{5}\tan u + C \\ &= \frac{1}{5}\tan 5x + C\end{aligned}$$

13. $u = 2 - \cos 2x \longrightarrow du = 2\sin 2x\,dx \longrightarrow \frac{1}{2}\,du = \sin 2x\,dx$

$$\begin{aligned}\int \sin 2x\sqrt[3]{2-\cos 2x}\,\mathrm{d}x &= \frac{1}{2}\int u^{1/3}\,\mathrm{d}u \\ &= \frac{1}{2}\,\frac{3}{4}u^{4/3} + C \\ &= \frac{3}{8}(2-\cos 2x)^{4/3} + C\end{aligned}$$

14. $u = \cos t \longrightarrow du = -\sin t\ dt \longrightarrow -du = \sin t\ dt$

$$\begin{aligned}\int \cos^2 t \sin t \,\mathrm{d}t &= -\int u^2 \,\mathrm{d}u \\ &= -\frac{1}{3}\, u^3 + C \\ &= -\frac{1}{3}\, \cos^3 t + C\end{aligned}$$

15. $u = x^3 + 3 \longrightarrow du = 3x^2\ dx \longrightarrow \frac{1}{3}\ du = x^2\ dx$
$x^3 = u - 3$

$$\begin{aligned}\int (x^3)^{1/4}\, x^5 \,\mathrm{d}x &= \int (x^3+3)^{1/4}\, x^3 x^2 \,\mathrm{d}x \\ &= \frac{1}{3}\int u^{1/4}(u-3)\,\mathrm{d}u \\ &= \frac{1}{3}\int (u^{5/4} - 3u^{1/4})\,\mathrm{d}u \\ &= \frac{1}{3}\left[\frac{4}{9}u^{9/4} - \frac{12}{5}u^{5/4}\right] + C \\ &= \frac{4}{27}(x^3+3)^{9/4} - \frac{4}{5}(x^3+3)^{5/4} + C\end{aligned}$$

16. $u = 1 + \cos x \longrightarrow du = -\sin x\ dx \longrightarrow -du = \sin x\ dx$

$$\begin{aligned}\int \frac{4\sin x}{(1+\cos x)^2}\,\mathrm{d}x &= -4\int u^{-2}\,\mathrm{d}u \\ &= (-4)(-1)u^{-1} + C \\ &= \frac{4}{1+\cos x} + C\end{aligned}$$

17. $u = \frac{1}{3}x \longrightarrow du = \frac{1}{3}\ dx \longrightarrow 3\ du = dx$

$$\begin{aligned}\int \sin\frac{1}{3}\,\mathrm{d}x &= 3\int \sin u\,\mathrm{d}u \\ &= -3\cos u + C \\ &= -3\cos\tfrac{1}{3}x + C\end{aligned}$$

18. $u = 4t^2 \longrightarrow du = 8t\ dt \longrightarrow \frac{1}{16}\ du = \frac{1}{2}t\ dt$

$$\begin{aligned}\int \frac{1}{2}t\cos 4t^2\,\mathrm{d}t &= \frac{1}{16}\int \cos u\,\mathrm{d}u \\ &= \frac{1}{16}\sin u + C \\ &= \frac{1}{16}\sin 4t^2 + C\end{aligned}$$

19. $u = x - 2 \longrightarrow du = dx$

$$\begin{aligned}
\int (x^2 - 4x + 4)^{4/3}\,\mathrm{d}x &= \int \left[(x-2)^2\right]^{4/3}\,\mathrm{d}x \\
&= \int u^{8/3}\,\mathrm{d}u \\
&= \frac{3}{11}u^{11/3} + C \\
&= \frac{3}{11}(x-2)^{11/3} + C
\end{aligned}$$

20. $u = x^3 + 3x^2 + 1 \longrightarrow du = (3x^2 + 6x)\,dx \longrightarrow \dfrac{1}{3}\,du = (x^2 + 2x)\,dx$

$$\begin{aligned}
\int \frac{x^2 + 2x}{\sqrt{x^3 + 3x^2 + 1}}\,\mathrm{d}x &= \frac{1}{3}\int u^{-1/2}\,\mathrm{d}u \\
&= \left(\frac{1}{3}\right)\left(2u^{1/2}\right) + C \\
&= \frac{2}{3}\sqrt{x^3 + 3x^2 + 1} + C
\end{aligned}$$