1.
$$\begin{aligned}
\frac{dy}{dx} &= y^2 \\
y^{-2}\,dy &= dx \\
\int y^{-2}\,\mathrm{d}y &= \int \mathrm{d}x \\
-y^{-1} &= x + C \\
-\frac{1}{y} &= x + C
\end{aligned}$$

2.
$$\begin{aligned}
y\frac{dy}{dx} &= x \\
y\,dy &= x\,dx \\
\int y\,\mathrm{d}y &= \int x\,\mathrm{d}x \\
\frac{1}{2}y^2 &= \frac{1}{2}x^2 + C \\
y^2 &= x^2 + D
\end{aligned}$$

3.
$$\begin{aligned}
x^2\frac{dy}{dx} + y &= 0 \\
x^2\frac{dy}{dx} &= -y \\
-y^{-1}\,dy &= x^{-2}\,dx \\
-\int y^{-1}\,\mathrm{d}y &= \int x^{-2}\,\mathrm{d}x \\
-\ln|y| &= -x^{-1} + C \\
\ln|y| &= \frac{1}{x} + D \\
y &= e^{(1/x)+D} \\
y &= e^{1/x}e^{D} \\
y &= Ae^{1/x}
\end{aligned}$$

4.
$$\begin{aligned}
\frac{du}{dt} &= e^{u+2t} \\
\frac{du}{dt} &= e^{u}e^{2t} \\
e^{-u}\ du &= e^{2t}\ dt \\
\int e^{-u}\,\mathrm{d}u &= \int e^{2t}\,\mathrm{d}t \\
-e^{-u} &= \frac{1}{2}e^{2t} + C \\
-2e^{-u} &= e^{2t} + D \\
\frac{-2}{e^{u}} &= e^{2t} + D
\end{aligned}$$

5.
$$\begin{aligned}
\frac{dy}{dx} &= \frac{x^2\sqrt{x^3-3}}{y^2} \\
y^2\ dy &= x^2\sqrt{x^3-3}\ dx \\
\int y^2\,\mathrm{d}y &= \int x^2\sqrt{x^3-3}\,\mathrm{d}x \\
\frac{1}{3}\ y^3 &= \frac{2}{9}\left(x^3-3\right)^{3/2} + C \text{ (Via substitution)} \\
3y^3 &= 2\left(x^3-3\right)^{3/2} + D
\end{aligned}$$

6.
$$\begin{aligned}
\frac{dy}{dx} &= \frac{\sqrt{x}+x}{\sqrt{y}-y} \\
(y^{1/2}-y)\ dy &= (x^{1/2}+x)\ dx \\
\int (y^{1/2}-y)\,\mathrm{d}y &= \int (x^{1/2}+x)\,\mathrm{d}x \\
\frac{2}{3}\ y^{3/2} - \frac{1}{2}\ y^2 &= \frac{2}{3}\ x^{3/2} + \frac{1}{2}\ x^2 + C \\
4y^{3/2} - 3y^2 &= 4x^{3/2} + 3x^2 + D
\end{aligned}$$

7.
$$\begin{aligned}
\frac{du}{dv} &= \frac{\cos 2v}{\sin 3u} \\
\sin 3u \; du &= \cos 2v \; dv \\
\int \sin 3u \, \mathrm{d}u &= \int \cos 2v \, \mathrm{d}v \\
-\frac{1}{3}\cos 3u &= \frac{1}{2}\sin 2v + C \\
-2\cos 3u &= 3\sin 2v + D
\end{aligned}$$

IMPORTANT NOTE:
When you need to antidifferentiate a trig function with a linear argument...

$\int \sin(Ax+B)\, \mathrm{d}x$

$u = Ax + B \;\longrightarrow\; \frac{1}{A}\; du = dx$

$\frac{1}{A}\int \sin u \, \mathrm{d}u = -\frac{1}{A}\cos u + C$

$\therefore\; \int \sin(Ax+B)\, \mathrm{d}x = -\frac{1}{A}\cos(Ax+B) + C$

It's a quick way to antidifferentiate trig functions with linear arguments!

Here's another example:

$\int \cos(9x+3)\, \mathrm{d}x = \frac{1}{9}\sin(9x+3) + C$

8. Note: We will let $\frac{d^2y}{dx^2} = \frac{dy'}{dx}$ and antidifferentiate twice.
$$\begin{aligned}
\frac{d^2y}{dx^2} &= 5x^2 + 1 \\
\frac{dy'}{dx} &= 5x^2 + 1 \\
dy' &= (5x^2+1)\; dx \\
\int \mathrm{d}y' &= \int (5x^2+1)\, \mathrm{d}x \\
y' &= \frac{5}{3}x^3 + x + C \\
\frac{dy}{dx} &= \frac{5}{3}x^3 + x + C \\
dy &= \left(\frac{5}{3}x^3 + x + C\right)\; dx \\
y &= \frac{5}{12}x^4 + \frac{1}{2}x^2 + Cx + D
\end{aligned}$$

9.
$$\begin{aligned}
\frac{dy}{dx} &= (x+2)(x+1) \\
\frac{dy}{dx} &= x^2+3x+2 \\
dy &= (x^2+3x+2)\ dx \\
\int \mathrm{d}y &= \int (x^2+3x+2)\,\mathrm{d}x \\
y &= \frac{1}{3}x^3+\frac{3}{2}x^2+2x+C
\end{aligned}$$

Since $y=-\frac{3}{2}$ when $x=-2$

$-\frac{3}{2}=\frac{1}{3}(-2)^3+\frac{3}{2}(-2)^2+2(-2)+C \longrightarrow C=-\frac{5}{6}$

$\therefore\ y=\frac{1}{3}x^3+\frac{3}{2}x^2+2x-\frac{5}{6}$

10. Note: We will again let $\frac{d^2y}{dx^2}=\frac{dy'}{dx}$ and antidifferentiate twice.

$$\begin{aligned}
\frac{d^2y}{dx^2} &= -3x^{-4} \\
\frac{dy'}{dx} &= -3x^{-4} \\
dy' &= -3x^{-4}\ dx \\
\int \mathrm{d}y' &= \int -3x^{-4}\,\mathrm{d}x \\
y' &= x^{-3}+C
\end{aligned}$$

Now, since $\frac{dy}{dx}=x^{-3}+C$ and $\frac{dy}{dx}=-1$ when $x=1 \longrightarrow C=-2$

$$\begin{aligned}
\frac{dy}{dx} &= x^{-3}-2 \\
dy &= (x^{-3}-2)\ dx \\
\int \mathrm{d}y &= \int (x^{-3}-2)\,\mathrm{d}x \\
y &= -\frac{1}{2}x^{-2}-2x+D
\end{aligned}$$

Since $y=\frac{1}{2}$ when $x=1 \longrightarrow D=3$

$\therefore y=-\frac{1}{2}x^{-2}-2x+3$

$\therefore y=-\frac{1}{2x^2}-2x+3$

11.
$$\begin{aligned} \frac{dy}{dx} &= 4x^3 y \\ \frac{1}{y}\, dy &= 4x^3\, dx \\ \int \frac{1}{y}\, \mathrm{d}y &= \int 4x^3\, \mathrm{d}x \\ \ln|y| &= x^4 + C \end{aligned}$$

Since $(0,7)$ is on the curve $\longrightarrow\; C = \ln 7$

$$\begin{aligned} \ln|y| &= x^4 + \ln 7 \\ y &= e^{x^4+\ln 7} \\ y &= e^{x^4} e^{\ln 7} \\ y &= 7e^{x^4} \end{aligned}$$

12.
$$\begin{aligned} \frac{dy}{dx} &= \frac{y^2}{x^3} \\ y^{-2}\, dy &= x^{-3}\, dx \\ \int y^{-2}\, \mathrm{d}y &= \int x^{-3}\, \mathrm{d}x \\ -y^{-1} &= -\frac{1}{2}\, x^{-2} + C \end{aligned}$$

Since $(1,1)$ is on the curve $\longrightarrow\; C = -\frac{1}{2} \;\longrightarrow\; \frac{1}{y} = \frac{1}{2x^2} + \frac{1}{2}$

13. $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$

$$e^{2y} dy = 3x^2 dx$$

$$\frac{1}{2}e^{2y} = x^3 + C \;\longrightarrow\; C = \frac{e}{2}$$

$$\frac{1}{2}e^{2y} = x^3 + \frac{e}{2}$$

$$e^{2y} = 2x^3 + e$$

$$2y = \ln(2x^3 + e)$$

$$y = \frac{1}{2}\ln(2x^3 + e)$$

14. $\dfrac{dy}{dx} = y^2(6-2x)$

$y^{-2}dy = (6-2x)dx$

$-y^{-1} = 6x - x^2 + C \longrightarrow C = -13$

$-\dfrac{1}{y} = 6x - x^2 - 13$

$y = \dfrac{1}{x^2 - 6x + 13}$

15. $\dfrac{dy}{dx} = -\dfrac{2x}{y}$

$y\ dy = -2x\ dx$

$\dfrac{1}{2}y^2 = -x^2 + C \longrightarrow C = \dfrac{3}{2}$

$\dfrac{1}{2}y^2 = -x^2 + \dfrac{3}{2}$

$y^2 = -2x^2 + 3$

$y = \pm\sqrt{3-2x^2}$ and since $f((1) = -1$

$y = -\sqrt{3-2x^2}$

16. $$\frac{dy}{dx} = \frac{1+y}{x}$$

$$\frac{1}{1+y}\ dy = \frac{1}{x}\ dx$$

$$\ln|1+y| = \ln|x| + C$$

$$1+y = e^{\ln|x|+C}$$

$$1+y = A|x| \ \longrightarrow\ A = 2$$

$$y = 2|x| - 1$$

17. $$\frac{dy}{dx} = xy^3$$

$$y^{-3}dy = xdx$$

$$-\frac{1}{2}y^{-2} = \frac{1}{2}x^2 + C$$

$$-\frac{1}{2y^2} = \frac{1}{2}x^2 + C \ \longrightarrow\ C = -\frac{5}{8}$$

$$-\frac{1}{2y^2} = \frac{1}{2}x^2 - \frac{5}{8}$$

$$\frac{4}{y^2} = 5 - 4x^2$$

18. $$\frac{dy}{dx} = \frac{y-1}{x^2}$$

$$\frac{1}{y-1}\ dy = x^{-2}dx$$

$$\ln|y-1| = -\frac{1}{x} + C$$

$$y-1 = e^{-\frac{1}{x}+C}$$

$$y-1 = Ae^{-\frac{1}{x}} \ \longrightarrow\ A = -\sqrt{e}$$

$$y = 1 - \sqrt{e}\ e^{-\frac{1}{x}}$$