1. $\int (12x^2 + 6x - 5)\,\mathrm{d}x = 4x^3 + 3x^2 - 5x + C$

2. $\int (6x^9 - 4x^7 + 3x^2 + 1)\,\mathrm{d}x = \frac{3}{5}x^{10} - \frac{1}{2}x^8 + x^3 + x + C$

3. $\int \left(\sqrt{x} + \sqrt[3]{x}\right)\mathrm{d}x = \frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$

4. $$\begin{aligned} \int \frac{6}{x^5}\,\mathrm{d}x &= -\frac{3}{2}x^{-4} + C \\ &= -\frac{3}{2x^4} + C \end{aligned}$$

5. $$\begin{aligned} \int \frac{t^3 + 2t^2}{\sqrt{t}}\,\mathrm{d}t &= \int t^{-1/2}(t^3 + 2t^2)\,\mathrm{d}t \\ &= \int (t^{5/2} + 2t^{3/2})\,\mathrm{d}t \\ &= \frac{2}{7}t^{7/2} + \frac{4}{5}t^{5/2} + C \end{aligned}$$

6. $\int (\sin x - 2\cos x)\,\mathrm{d}x = -\cos x - 2\sin x + C$

7. $\int \sec^2 t + t^2\,\mathrm{d}t = \tan t + \frac{1}{3}t^3 + C$

8. $$\begin{aligned} \int \frac{x^2 + x + 1}{x}\,\mathrm{d}x &= \int x^{-1}(x^2 + x + 1)\,\mathrm{d}x \\ &= \int (x + 1 + x^{-1})\,\mathrm{d}x \\ &= \frac{1}{2}x^2 + x + \ln|x| + C \end{aligned}$$

9. $\int (x^2 + x^3)\,\mathrm{d}x = \frac{1}{3}x^3 + \frac{1}{4}x^4 + C \;\longrightarrow\; f'(x) = \frac{1}{3}x^3 + \frac{1}{4}x^4 + C$

   $\int \left(\frac{1}{3}x^3 + \frac{1}{4}x^4 + C\right)\mathrm{d}x = \frac{1}{12}x^4 + \frac{1}{20}x^5 + Cx + D$

   $\therefore\; f(x) = \frac{1}{12}x^4 + \frac{1}{20}x^5 + Cx + D$

10. $\int \left(3x^{1/2} - x^{-1/2}\right)\mathrm{d}x = 2x^{3/2} - 2x^{1/2} + C$

    Since $f(1) = 2 \;\longrightarrow\; 2 = 2(1)^{3/2} - 2(1)^{1/2} + C \;\longrightarrow\; C = 2$

    $\therefore\; f(x) = 2x^{3/2} - 2x^{1/2} + 2$

11. $\int 2x^{-1}\,\mathrm{d}x = 2\ln|x| + C$

Since $f(-1) = 7 \longrightarrow 7 = 2\ln|-1| + C \longrightarrow C = 7$

$\therefore\ f(x) = 7 + 2\ln|x|$

12. $\int \left(x + x^{1/2}\right)\mathrm{d}x = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + C$

Since $f'(1) = 2 \longrightarrow 2 = \frac{1}{2}(1)^2 + \frac{2}{3}(1)^{3/2} + C \longrightarrow C = \frac{5}{6}$

$\therefore\ f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + \frac{5}{6}$

$\int \left(\frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + \frac{5}{6}\right)\mathrm{d}x = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x + D$

Since $f(1) = 1 \longrightarrow 1 = \frac{1}{6}(1)^3 + \frac{4}{15}(1)^{5/2} + \frac{5}{6}(1) + D \longrightarrow D = -\frac{4}{15}$

$\therefore\ f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x - \frac{4}{15}$

13. $\int 3x^{-2}\,\mathrm{d}x = -3x^{-1} + C$

If $x > 0$ use $f(1) = 0 \longrightarrow C = 3$
If $x < 0$ use $f(-1) = 0 \longrightarrow C = -3$

$$\therefore\ f(x) = \begin{cases} -\frac{3}{x} + 3 & x > 0 \\ -\frac{3}{x} - 3 & x < 0 \end{cases}$$

14. $\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x = \sin^{-1}x + C$

15. $\int (x + \sec x \tan x)\,\mathrm{d}x = \frac{1}{2}x^2 + \sec x + C$

16. $$\begin{aligned} \int \left(\frac{3}{x^2} + \frac{5}{x^4}\right)\mathrm{d}x &= \int \left(3x^{-2} + 5x^{-4}\right)\mathrm{d}x \\ &= -3x^{-1} - \frac{5}{3}x^{-3} + C \\ &= -\frac{3}{x} - \frac{5}{3x^3} + C \end{aligned}$$

17. Since $(1,6)$ is on f, we know that $f(1)=6$.

We also know that the slope of a tangent to f is $2x+1 \longrightarrow f'(x)=2x+1$.

$$\int (2x+1)\,\mathrm{d}x = x^2+x+C$$

Since $f(1)=6 \longrightarrow 6=(1)^2+1+C \longrightarrow C=4$

$\therefore f(x)=x^2+x+4 \longrightarrow f(2)=10$.

18.
$$\begin{aligned}\int \frac{\sin x}{\cos^2 x}\,\mathrm{d}x &= \int \frac{1}{\cos x}\,\frac{\sin x}{\cos x}\,\mathrm{d}x \\ &= \int \sec x \tan x\,\mathrm{d}x \\ &= \sec x + C\end{aligned}$$

19. Since $(3,2)$ is on f, we know that $f(3)=2$.

We also know that the slope of a tangent to f is $2x-3 \longrightarrow f'(x)=2x-3$.

$$\int (2x-3)\,\mathrm{d}x = x^2-3x+C$$

Since $f(3)=2 \longrightarrow 2=(3)^2-3(3)+C \longrightarrow C=2$

$\therefore y=x^2-3x+2$

20.
$$\begin{aligned}\frac{d^2y}{dx^2} &= 1-x^2 \\ \int (1-x^2)\,\mathrm{d}x &= x-\frac{1}{3}x^3+C \\ \frac{dy}{dx} &= x-\frac{1}{3}x^3+C\end{aligned}$$

We also know that the equation of the tangent to the curve at $(1,1)$ is $y=2-x$.

Since the slope of this line is -1, we know the value of the derivative, $\frac{dy}{dx}$, the slope of the tangent, is the same as the slope of the line thus $\frac{dy}{dx}=-1$.

$$\frac{dy}{dx}=-1 \longrightarrow -1=(1)-\frac{1}{3}(1)^3+C \longrightarrow C=-\frac{5}{3} \longrightarrow \frac{dy}{dx}=x-\frac{1}{3}x^3-\frac{5}{3}$$

$$\int \left(x-\frac{1}{3}x^3-\frac{5}{3}\right)\mathrm{d}x = \frac{1}{2}x^2-\frac{1}{12}x^4-\frac{5}{3}x+D$$

Since $(1,1)$ is on the curve, $1=\frac{1}{2}(1)^2-\frac{1}{12}(1)^4-\frac{5}{3}(1)+D \longrightarrow D=\frac{9}{4}$.

$\therefore\ y=\frac{1}{2}x^2-\frac{1}{12}x^4-\frac{5}{3}x+\frac{9}{4}$