1. Verification
   f is continuous on $[-1,1]$ and since $f'(x) = 3x^2 - 1$, $f' \exists \forall x \in (-1,1) \therefore f$ is differentiable on $(-1,1)$ and since $f(-1) = f(1) = 0$, Rolle's theorem holds for f on the given interval.

   Finding c
   $f'(x) = 0$ when $3x^2 - 1 = 0 \rightarrow x = \dfrac{1}{\sqrt{3}}$ or $x = -\dfrac{1}{\sqrt{3}}$
   Since both of these values are in $(-1,1)$, $c = \dfrac{1}{\sqrt{3}}$ or $c = -\frac{1}{\sqrt{3}}$

2. Verification
   f is continuous on $[0,2\pi]$ and since $f'(x) = -2\sin 2x$, $f' \exists \forall x \in (0,2\pi) \therefore f$ is differentiable on $(0,2\pi)$ and since $f(0) = f(2\pi) = 1$, Rolle's theorem holds for f on the given interval.

   Finding c
   $f'(x) = 0$ when $-2\sin 2x = 0 \rightarrow x = 0$ or $x = \dfrac{\pi}{2}$ or $x = \pi$ or $x = \frac{3\pi}{2}$ or $x = 2\pi$
   Taking only those values in $(0,2\pi)$, $c = \dfrac{\pi}{2}$ or $c = \pi$ or $c = \dfrac{3\pi}{2}$

3. $f(-1) = f(1) = 0$ but since $f'(x) = -\dfrac{2}{3\sqrt[3]{x}}$, f' can never be zero because $2 \neq 0$.
   This does not contradict Rolle's theorem because f' does not exist at $x = 0$ and therefore is not differentiable on $(-1,1)$ and so Rolle's theorem does not hold.

4. Connect the endpoints of sketch with a line segment. Then use a straightedge to draw lines that are tangent to the curve and parallel to the segment. Estimate the x-coordinate of the tangent points. All your tangents should have a slope of approximately $\dfrac{5}{9}$.

5. Verification
   f is continuous on $[0,3]$ and since $f'(x) = -2x$, $f' \exists \forall x$ and so f is differentiable on $(0,3) \therefore$ the Mean Value Theorem holds for f on the given interval.

   Finding c
   $f'(c) = -2c$ and $\dfrac{f(3) - f(0)}{3 - 0} = -3 \rightarrow -2x = -3 \rightarrow c = \dfrac{3}{2}$.

6. Verification
   f is continuous on $[1,2]$ and since $f'(x) = -\dfrac{1}{x^2}$, $f' \exists \forall x \in (1,2)$ and so f is differentiable on $(1,2) \therefore$ the Mean Value Theorem holds for f on the given interval.

   Finding c
   $f'(c) = -\dfrac{1}{c^2}$ and $\dfrac{f(2) - f(1)}{2 - 1} = -\dfrac{1}{2} \rightarrow -\dfrac{1}{c^2} = -\dfrac{1}{2} \rightarrow c = \sqrt{2}$ or $c = -\sqrt{2}$ but $-\sqrt{2}$ is not in $(1,2) \therefore c = \sqrt{2}$.

7. Verification
   f is continuous on $[2,9]$ and since $f'(x) = \dfrac{1}{3\sqrt[3]{(x-1)^2}}$, $f' \exists \forall x \in (2,9)$ and so f is differentiable on $(2,9) \therefore$ the Mean Value Theorem holds for f on the given interval.

   Finding c
   $f'(c) = \dfrac{1}{3\sqrt[3]{(c-1)^2}}$ and $f(9) - f(2)9 - 2 = \frac{1}{7} \rightarrow c = 4.564$ or $c = -2.564$ but $c = -2.564$ is not in $(2,9) \therefore c = 4.564$.

8. $f(x) = \begin{cases} x-1 & x \geq 1 \\ 1-x & x < 1 \end{cases} \rightarrow f'(x) = \begin{cases} 1 & x > 1 \\ -1 & x < 1 \end{cases}$

Now, for $a = a$ and $b = 3$, $\dfrac{f(b) - f(a)}{b - a} = \dfrac{1}{3}$ but $f'(x)$ is never equal to $\dfrac{1}{3}$.

This does not contradict the Mean Value Theorem because f is not differentiable on $(0, 3)$ since $f'(1) \nexists$.