

1. a $\rightarrow$ absolute minimum
b $\rightarrow$ relative maximum
c $\rightarrow$ relative minimum
d $\rightarrow$ relative maximum
e $\rightarrow$ relative minimum
f $\rightarrow$ relative maximum
g $\rightarrow$ relative minimum
h $\rightarrow$ absolute maximum
2. No relative extrema.
Absolute minimum of -1 at $x = -1$
No absolute maximum.
3. No relative extrema.
No absolute extrema.
4. No relative extrema.
Absolute maximum of 1 at $x = 0$.
Absolute minimum of 0 at $x = 1$.
5. Absolute maximum of 1 at $x = 0$.
Absolute minimum of -3 at $x = -2$.
No relative minimum at $x = 1$ since f does not exist to the right of $x = 1$.
6. No extrema.
7. Absolute maximum of 1 at $x = \frac{\pi}{2}$ and $x = -\frac{3\pi}{2}$
Absolute minimum of -1 at $x = \frac{3\pi}{2}$ and $x = -\frac{\pi}{2}$
8. No extrema
9. Absolute minimum of 0 at $x = 0$ and $x = 2$
No absolute maximum.
No relative extrema.
10. $f'(x) = 2 - 6x$
 - $f' \exists \forall x$
 - $f'(x) = 0$ when $x = \frac{1}{3}$. $\therefore x = \frac{1}{3}$ is the only critical number.
11. $f'(x) = 3x^2 - 3$
 - $f' \exists \forall x$
 - $f'(x) = 0$ when $x = 1$ or $x = -1$. $\therefore x = 1$ and $x = -1$.
12. $f'(x) = 6t^2 + 6t + 6$
 - $f' \exists \forall x$
 - $f'(x) \neq 0$ $\therefore$ no critical numbers.

13. $s'(t) = 6t^2 + 6t - 6$

- $s' \exists \forall t$

- $s'(t) = 0$ when $t^2 + t - 1 = 0 \rightarrow t = \frac{-1 + \sqrt{5}}{2}$ or $t = \frac{-1 - \sqrt{5}}{2}$

$\therefore$ critical numbers are $t = \frac{-1 + \sqrt{5}}{2}$ and $t = \frac{-1 - \sqrt{5}}{2}$

14. $g'(x) = \frac{1}{9\sqrt[9]{x^8}}$

- $g' \nexists$ at $x = 0$

- $g'(x)$ never zero

$\therefore$ critical number is $x = 0$. (Note: zero is in the domain of g .)

15. $g'(t) = \frac{10 + 5t}{3\sqrt[3]{t}}$

- $g' \nexists$ at $t = 0$

- $g'(t) = 0$ when $t = -2$

$\therefore$ critical numbers are $t = 0$ and $t = -2$

16. $f'(r) = \frac{1 - r^2}{(r^2 + 1)^2}$

- $f' \exists \forall r$ because $r^2 + 1 \neq 0$

- $f'(r) = 0$ when $1 - r^2 = 0 \rightarrow r = 1$ or $r = -1$

$\therefore$ critical numbers are $r = 1$ and $r = -1$

17. $f'(x) = \frac{2(x - 4)(7x - 8)}{5\sqrt[5]{x}}$

- $f' \nexists$ at $x = 0$

- $f'(x) = 0$ when $(x - 4)(7x - 8) = 0 \rightarrow x = 4$ or $x = \frac{8}{7}$

$\therefore$ critical numbers are $x = 4$ and $x = \frac{8}{7}$ and $x = 0$

18. $f'(x) = 2 \cos 2x$

- $f' \exists \forall x \in [0, 2\pi]$

- $f'(x) = 0$ when $x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$ or $x = \frac{5\pi}{4}$ or $x = \frac{7\pi}{4}$

$\therefore$ critical numbers are $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$ and $x = \frac{5\pi}{4}$ and $x = \frac{7\pi}{4}$

19. $f'(x) = 1 + \ln x$

- $f' \exists \forall x > 0$

- $f'(x) = 0$ when $1 + \ln x = 0 \rightarrow x = \frac{1}{e}$

$\therefore$ critical number is $x = \frac{1}{e}$

20. $f'(x) = 2x - 2$
 $f' \exists \forall x$
in $[0, 3]$ and $f'(x) = 0$ when $x = 1 \rightarrow$ only critical number is $x = 1$.
Since $f(0) = 2$, $f(3) = 5$ and $f(1) = 1$, by the Extreme Value Theorem f has an absolute maximum of 5 at $x = 3$ and an absolute minimum of 1 at $x = 1$.
21. $f'(x) = 3x^2 - 12$
 $f' \exists \forall x \in [-3, 5]$ and $f'(x) = 0$ when $x = 2$ or $x = -2 \rightarrow$ critical numbers are $x = 2$ and $x = -2$.
Since $f(-3) = 10$, $f(5) = 66$, $f(2) = -15$ and $f(-2) = 17$, by the Extreme Value Theorem f has an absolute maximum of 66 at $x = 5$ and an absolute minimum of -15 at $x = 2$.
22. $f'(x) = 6x^2 + 6x$
 $f' \exists \forall x \in [-2, 1]$ and $f'(x) = 0$ when $6x(x + 1) = 0 \rightarrow x = 0$ or $x = -1 \therefore$ critical numbers are $x = 0$ and $x = -1$.
Since $f(-2) = 0$, $f(1) = 9$, $f(0) = 4$ and $f(-1) = 5$, by the Extreme Value Theorem f has an absolute maximum of 9 at $x = 1$ and an absolute minimum of 0 at $x = -2$.
23. $f'(x) = 4x^3 - 8x$
 $f' \exists \forall x \in [-3, 2]$ and $f'(x) = 0$ when $4x(x^2 - 2) = 0 \rightarrow x = 0$ or $x = \sqrt{2}$ or $x = -\sqrt{2} \therefore$ critical numbers are $x = 0$ and $x = \sqrt{2}$ and $x = -\sqrt{2}$.
Since $f(-3) = 47$, $f(2) = 2$, $f(0) = 2$ and $f(\sqrt{2}) = -2$ and $f(-\sqrt{2}) = -2$, by the Extreme Value Theorem f has an absolute maximum of 47 at $x = -3$ and an absolute minimum of -2 at $x = \sqrt{2}$ and $x = -\sqrt{2}$.
24. $f'(x) = \frac{2x^3 - 2}{x^2}$
 $f' \nexists$ at $x = 0$ but $x = 0$ is not in $[\frac{1}{2}, 2]$ $\therefore x = 0$ is not a critical number. $f'(x) = 0$ when $2x^3 - 2 = 0 \rightarrow x = 1 \therefore x = 1$ is the only critical number.
Since $f(\frac{1}{2}) = \frac{17}{4}$, $f(2) = 5$ and $f(1) = 3$, by the Extreme Value Theorem f has an absolute maximum of 5 at $x = 2$ and an absolute minimum of 3 at $x = 1$.
25. $f'(x) = \frac{4}{5\sqrt[5]{x}}$
 $f' \nexists$ at $x = 0$ and $f'(x) \neq 0 \therefore$ only critical number is $x = 0$.
Since $f(-32) = 16$ and $f(1) = 1$ and $f(0) = 0$, by the Extreme Value Theorem f has an absolute maximum of 16 at $x = -32$ and an absolute minimum of 0 at $x = 0$.
26. $f'(x) = \frac{1 - x}{e^x}$
 $f' \exists \forall x \in [0, 1]$ and $f'(x) = 0$ when $x = 1 \therefore x = 1$ is the only critical number.
Since $f(0) = 0$ and $f(1) = \frac{1}{e}$, by the Extreme Value Theorem f has an absolute maximum of $\frac{1}{e}$ at $x = 1$ and an absolute minimum of 0 at $x = 0$.
27. $f'(x) = \frac{1 - \ln x}{x^2}$
 $f' \exists \forall x \in [1, 3]$ and $f'(x) = 0$ when $x = e \therefore$ the only critical number is $x = e$.
Since $f(1) = 0$ and $f(3) = \frac{\ln 3}{3}$ and $f(e) = \frac{1}{e}$, by the Extreme Value Theorem f has an absolute maximum of $\frac{1}{e}$ at $x = e$ and an absolute minimum of 0 at $x = 1$.
28. Answers may vary. A simple linear graph like $f(x) = x$ with a hole at some x between 0 and 1 would do. Below is just one example.

