1. f is increasing on $(-4.250, 1) \cup (3, \infty)$.
   f is decreasing on $(-\infty, -4.250) \cup (1, 3)$.
   f has a relative maximum at $x = 1$.
   f has a relative minimum of at $x = -4.250$ and $x = 3$.

2. $f'(x) = -1 - 2x$

   Critical numbers:
   $f' \exists \forall x$.
   $f'(x) = 0$ when $x = -\dfrac{1}{2}$.

   Analysis:
   $f'(x) > 0$ on $\left(-\infty, -\dfrac{1}{2}\right) \therefore f$ is increasing on $\left(-\infty, -\dfrac{1}{2}\right)$.
   $f'(x) < 0$ on $\left(-\frac{1}{2}, \infty\right) \therefore f$ is decreasing on $\left(-\dfrac{1}{2}, \infty\right)$.
   Since $f'(x) > 0$ on $\left(-\infty, -\dfrac{1}{2}\right)$ and $f'(x) < 0$ on $\left(-\dfrac{1}{2}, \infty\right)$ f has a relative maximum of $f\left(-\dfrac{1}{2}\right) = \dfrac{81}{4}$ at $x = -\dfrac{1}{2}$.

3. $f'(x) = 3x^2 + 1$

   Critical numbers:
   $f' \exists \forall x$.
   $f'(x) \neq 0$
   $\therefore$ no critical numbers.

   Analysis:
   Since $f'(x) > 0 \forall x f$ is always increasing and has no extrema.

4. $f'(x) = 4x - 4x^3$

   Critical numbers:
   $f' \exists \forall x$.
   $f'(x) = 0$ when $4x(1 - x^2) = 0 \to x = 0$ or $x = 1$or $x = -1$.

   Analysis:
   $f'(x) > 0$ on $(-\infty, -1) \cup (0, 1) \therefore f$ is increasing on $(-\infty, -1) \cup (0, 1)$.
   $f'(x) < 0$ on $(-1, 0) \cup (1, \infty) \therefore f$ is decreasing on $(-1, 0) \cup (1, \infty)$.

   $f'(x) > 0$ on $(-\infty, -1)$ and $f'(x) < 0$ on $(-1, 0) \therefore f$ has a relative maximum of $f(-1) = 1$ at $x = -1$.
   $f'(x) < 0$ on $(-1, 0)$ and $f'(x) > 0$ on $(0, 1) \therefore f$ has a relative minimum of $f(0) = 0$ at $x = 0$.
   $f'(x) > 0$ on $(0, 1)$ and $f'(x) < 0$ on $(1, \infty) \therefore f$ has a relative maximum of $f(1) = 1$ at $x = 1$.

5. $f'(x) = x^2(x-4)^3(7x-12)$

Critical numbers:
$f' \exists \forall x$.
$f'(x) = 0$ when $x^2(x-4)^3(7x-12) = 0 \rightarrow x = 0$ or $x = 4$ or $x = \dfrac{12}{7}$.

You may want to use a chart on this problem. Remember ... the chart is data for you to use. You cannot refer to it in your answer.

Analysis:
$f'(x) > 0$ on $(-\infty, 0) \cup \left(0, \dfrac{12}{7}\right) \cup (4, \infty) \;\therefore\; f$ is increasing on $(-\infty, 0) \cup \left(0, \dfrac{12}{7}\right) \cup (4, \infty)$.
$f'(x) < 0$ on $\left(\dfrac{12}{7}, 4\right) \;\therefore\; f$ is decreasing on $\left(\dfrac{12}{7}, 4\right)$.
$f'(x) > 0$ on $\left(0, \dfrac{12}{7}\right)$ and $f'(x) < 0$ on $\left(\dfrac{12}{7}, 4\right) \;\therefore\; f$ has a relative maximum of $f\left(\dfrac{12}{7}\right) = 137.511$ at $x = \dfrac{12}{7}$.
$f'(x) < 0$ on $\left(\dfrac{12}{7}, 4\right)$ and $f'(x) > 0$ on $(4, \infty) \;\therefore\; f$ has a relative minimum of $f(4) = 0$ at $x = 4$.

6. $f'(x) = \dfrac{6x+1}{5\sqrt[5]{x^4}}$

Critical numbers:
$f' \nexists$ at $x = 0$.
$f'(x) = 0$ when $x = -\dfrac{1}{6}$.

You may want to use a chart on this problem. Remember ... the chart is data for you to use. You cannot refer to it in your answer.

Analysis:
$f'(x) > 0$ on $\left(-\dfrac{1}{6}, 0\right) \cup (0, \infty) \;\therefore\; f$ is increasing on $\left(-\dfrac{1}{6}, 0\right) \cup (0, \infty)$.
$f'(x) < 0$ on $\left(-\infty, -\dfrac{1}{6}\right) \;\therefore\; f$ is decreasing on $\left(-\infty, -\dfrac{1}{6}\right)$.
$f'(x) < 0$ on $\left(-\infty, -\dfrac{1}{6}\right)$ and $f'(x) > 0$ on $\left(-\dfrac{1}{6}, 0\right) \;\therefore\; f$ has a relative minimum of $f\left(-\dfrac{1}{6}\right) = -.582$ at $x = -\dfrac{1}{6}$.

7. $f'(x) = \dfrac{3x - 4x^2}{2\sqrt{x - x^2}}$

Note: The domain of f is $[0, 1]$ so any chart you may use must begin at $x = 0$ and end at $x = 1$.

Critical numbers:
$f' \exists \forall x$ in $(0, 1)$.
$f'(x) = 0$ when $3x - 4x^2 = 0 \rightarrow x = 0$ or $x = \dfrac{3}{4}$.

Analysis:
$f'(x) > 0$ on $\left(0, \dfrac{3}{4}\right) \rightarrow f$ is increasing on $\left(0, \dfrac{3}{4}\right)$.
$f'(x) < 0$ on $\left(\dfrac{3}{4}, 1\right) \rightarrow f$ is decreasing on $\left(\dfrac{3}{4}, 1\right)$.
$f'(x) > 0$ on $\left(0, \dfrac{3}{4}\right)$ and $f'(x) < 0$ on $\left(\dfrac{3}{4}, 1\right) \rightarrow f$ has a relative maximum of $\dfrac{3\sqrt{3}}{16}$ at $x = \dfrac{3}{4}$.

8. $f'(x) = 1 - 2\cos x$

Critical numbers:
$f' \exists \forall x$ in $[0, 2\pi]$.
$f'(x) = 0$ when $2\cos x = 1 \; \rightarrow \; x = \dfrac{\pi}{3}$ or $x = \dfrac{5\pi}{3}$.

Analysis:
$f'(x) > 0$ on $\left(\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right) \; \rightarrow \; f$ is increasing on $\left(\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right)$.
$f'(x) < 0$ on $\left(0, \dfrac{\pi}{3}\right) \cup \left(\dfrac{5\pi}{3}, 2\pi\right) \rightarrow f$ is decreasing on $\left(0, \dfrac{\pi}{3}\right) \cup \left(\dfrac{5\pi}{3}, 2\pi\right)$
$f'(x) < 0$ on $\left(0, \dfrac{\pi}{3}\right)$ and $f'(x) > 0$ on $\left(\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right) \; \rightarrow \; f$ has a relative minimum of $\dfrac{\pi - 3\sqrt{3}}{3}$ at $x = \dfrac{\pi}{3}$.
$f'(x) > 0$ on $\left(\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right)$ and $f'(x) < 0$ on $\left(\dfrac{5\pi}{3}, 2\pi\right) \; \rightarrow \; f$ has a relative maximum of $\dfrac{5\pi + 3\sqrt{3}}{3}$ at $x = \dfrac{5\pi}{3}$.

9. $f'(x) = 3x^2 + 4x - 1$

Critical numbers:
$f' \exists \forall x$.
$f'(x) = 0$ when $x = \dfrac{-2 + \sqrt{7}}{3}$ or $x = \dfrac{-2 - \sqrt{7}}{3}$.

Analysis:
$f'(x) > 0$ on $\left(-\infty, \dfrac{-2 - \sqrt{7}}{3}\right) \cup \left(\dfrac{-2 + \sqrt{7}}{3}, \infty\right) \; \rightarrow \; f$ is increasing on $\left(-\infty, \dfrac{-2 - \sqrt{7}}{3}\right) \cup \left(\dfrac{-2 + \sqrt{7}}{3}, \infty\right)$.
$f'(x) < 0$ on $\left(\dfrac{-2 - \sqrt{7}}{3}, \dfrac{-2 + \sqrt{7}}{3}\right) \; \rightarrow \; f$ is decreasing on $\left(\dfrac{-2 - \sqrt{7}}{3}, \dfrac{-2 + \sqrt{7}}{3}\right)$.

10. $f'(x) = 6x^5 + 192$

Critical numbers:
$f' \exists \forall x$.
$f'(x) = 0$ when $6x^5 + 192 = 0 \; \rightarrow \; x = -2$.

Analysis:
$f'(x) < 0$ on $(-\infty, -2) \rightarrow \; f$ is decreasing on $(-\infty, -2)$.
$f'(x) > 0$ on $(-2, \infty) \rightarrow \; f$ is increasing on $(-2, \infty)$.

11. $f'(x) = xe^x + e^x$

Critical numbers:
$f' \exists \forall x$.
$f'(x) = 0\, textrmwhen\, e^x(x + 1) = 0 \; \rightarrow \; x = -1$.

Analysis:
$f'(x) < 0$ on $(-\infty, -1) \; \rightarrow \; f$ is decreasing on $(-\infty, -1)$.
$f'(x) > 0$ on $(-1, \infty) \; \rightarrow \; f$ is increasing on $(-1, \infty)$.

12. $f'(x) = \dfrac{2 - \ln x}{2\sqrt{x^3}}$

    Critical numbers:
    $f' \nexists$ at $x = 0$. (Zero is not in the domain of f so it is not a critical number but we will include it in our analysis.
    $f'(x) = 0$ when $2 - \ln x = 0 \rightarrow x = e^2$.

    Analysis:
    $f'(x) > 0$ on $(0, e^2) \rightarrow f$ is increasing on $(0, e^2)$.
    $f'(x) < 0$ on $(e^2, \infty) \rightarrow f$ is decreasing on (e^2, ∞).

13. $f'(x) = \dfrac{-1 + 2\sqrt{1-x}}{2\sqrt{1-x}}$

    Critical numbers:
    $f' \nexists$ at $x = 1$
    $f'(x) = 0$ when $-1 + 2\sqrt{1-x} = 0 \rightarrow x = \dfrac{3}{4}$.

    This critical number is in $[0, 1]$.

    Analysis:
    Since f is continuous on $[0, 1]$ and has only one critical number in $[0, 1]$, by the Extreme Value Theorem f has an absolute maximum of $\frac{5}{4}$ at $x = \frac{3}{4}$ and an absolute minimum of 1 at $x = 0$ (or $x = 1$).
    f has no other extrema. (The relative maximum which occurs at $x = \frac{3}{4}$ is an absolute maximum and so is not listed as a relative extrema.

14. $g'(x) = \dfrac{1 - x^2}{(x^2 + 1)^2}$

    Critical numbers:
    $g' \exists \forall x$ since $x^2 + 1 \neq 0$
    $g'(x) = 0$ when $x^2 - 1 = 0 \rightarrow x = 1$ or $x = -1$
    Both critical numbers are in $[-5, 5]$

    Analysis:
    Since g is continuous on $[-5, 5]$, g has an absolute minimum of $-\frac{1}{2}$ at $x = -1$ and an absolute maximum of $\frac{1}{2}$ at $x = 1$.

15. Your graph should be increasing and concave down on $(-\infty, 1)$ and decreasing on $(1, \infty)$. It should have a "stepdown" at the point $(4, 2)$. It should also be concave down on the interval $(4, \infty)$ and there should be an inflection point between $x = 1$ and $x = 4$.

16. Your graph should have a vertical asymptote at $x = 3$. Before $x = 3$ your graph should be decreasing and concave down. The graph should be increasing on $(3, 5)$, decreasing on $(5, \infty)$ with a relative maximum at $x = 5$.