These are answers only!
Make sure that when you do these problems you use our normal justifications!

1. Vertical asymptotes: None.
   Horizontal asymptotes: None.
   Oblique asymptotes: None.

   $f'(x) = -3x^2 + 10x - 3$

   Critical numbers:
   $f' \exists \forall x$
   $f'(x) = 0$ when $x = \dfrac{1}{3}$ or $x = 3$.

   Conclusions:
   f is increasing on $\left(\dfrac{1}{3}, 3\right)$.
   f is decreasing on $\left(\infty, \dfrac{1}{3}\right) \cup (3, \infty)$.
   f has a relative minimum of $\dfrac{14}{27}$ at $x = \dfrac{1}{3}$.
   f has a relative maximum of 10 at $x = 3$.

   $f''(x) = 10 - 6x$

   Possible inflection points:
   $f'' \exists \forall x$
   $f''(x) = 0$ when $x = \dfrac{3}{5}$.

   Conclusions:
   f is concave up on $\left(-\infty, \dfrac{5}{3}\right)$.
   f is concave down on $\left(\dfrac{5}{3}, \infty\right)$.
   f has an inflection point at $\left(\dfrac{5}{3}, \dfrac{142}{27}\right)$.

2. Vertical asymptotes: None.
   Horizontal asymptotes: None.
   Oblique asymptotes: None.

   $f'(x) = 4x^3 - 12x$

   Critical numbers:
   $f' \exists \forall x$
   $f'(x) = 0$ when $x = 0$ or $x = \sqrt{3}$ or $x = -\sqrt{3}$.

   Conclusions:
   f is increasing on $\left(-\sqrt{3}, 0\right) \cup \left(\sqrt{3}, \infty\right)$.
   f is decreasing on $\left(-\infty, -\sqrt{3}\right) \cup (0, \sqrt{3})$.

   f has a relative minimum of -9 at $x = -\sqrt{3}$ and -9 at $x = \sqrt{3}$.

f has a relative maximum of 0 at $x = 0$.

f has an absolute minimum of -9 at both $x = \sqrt{3}$ and $x = \sqrt{3}$.

$f''(x) = 12x^2 - 12$

Possible inflection points:
$f'' \exists \forall x$
$f''(x) = 0$ when $x = 1$ or $x = -1$.

Conclusions:
f is concave up on $(-\infty, -1) \cup (1, \infty)$.
f is concave down on $(-1, 1)$.

f has inflection points at $(-1, -5)$ and $(1, -5)$.

3. Vertical asymptotes: $x = 1$.
Horizontal asymptotes: $y = 1$.
Oblique asymptotes: None.

$$f'(x) = \frac{1}{(x-1)^2}$$

Critical numbers:
$f' \nexists$ at $x = 1$ but $x = 1$ not in f so it is not a critical number .
$f'(x) \neq 0$

Conclusions:
f is decreasing on $(-\infty, 1) \cup (1, \infty)$.

f has no relative extrema.
f has no absolute extrema.

$$f''(x) = \frac{2}{(x-1)^3}$$

Possible inflection points:
$f'' \nexists$ when $x = 1$
$f''(x) \neq 0$

Conclusions:
f is concave up on $(1, \infty)$.
f is concave down on $(-\infty, 1)$.

f has no inflection points .

4. Vertical asymptotes: $x = 3$ and $x = -3$
Horizontal asymptotes: $y = 0$
Oblique asymptotes: None

$$\frac{dy}{dx} = \frac{-2x}{(x^2-9)^2}$$

Critical numbers:
$\frac{dy}{dx} \nexists$ when $x = 3$ or $x = -3$. (Neither are critical numbers but will be included in our analysis as usual.)
$\frac{dy}{dx}$ = 0 when x=0.

Conclusions:
$y = \dfrac{1}{x^2 - 9}$ is increasing on $(-\infty, -3) \cup (-3, 0)$.
$y = \dfrac{1}{x^2 - 9}$ is decreasing on $(0, 3) \cup (3, \infty)$.
$y = \dfrac{1}{x^2 - 9}$ has a relative maximum of $-\dfrac{1}{9}$ at $x = 0$.

$$\frac{d^2y}{dx^2} = \frac{6(x^2 + 3)}{(x^2 - 9)^3}$$

Possible inflection points:
$\dfrac{d^2y}{dx^2} \nexists$ when $x = 3$ or $x = -3$.
$\dfrac{d^2y}{dx^2} \neq 0$.

$y = \dfrac{1}{x^2 - 9}$ is concave up on $(-\infty, -3) \cup (3, \infty)$.
$y = \dfrac{1}{x^2 - 9}$ is concave down on $(-3, 3)$.
$y = \dfrac{1}{x^2 - 9}$ has no inflection points.

5. Vertical asymptotes: $x = 1$ and $x = -2$
Horizontal asymptotes: $y = 0$
Oblique asymptotes: None

$$\frac{dy}{dx} = \frac{-2x - 1}{(x - 1)^2 (x + 2)^2}$$

Critical numbers:
$\dfrac{dy}{dx} \nexists$ when $x = 1$ or $x = -2$. (Neither are critical numbers but will be included in our analysis as usual.)
$\dfrac{dy}{dx}$ = 0 when x=-$\dfrac{1}{2}$.

Conclusions:
$y = \dfrac{1}{(x - 1)(x + 2)}$ is increasing on $(-\infty, -2) \cup \left(-2, -\dfrac{1}{2}\right)$.
$y = \dfrac{1}{(x - 1)(x + 2)}$ is decreasing on $\left(-\dfrac{1}{2}, 1\right) \cup (1, \infty)$.
$y = \dfrac{1}{(x - 1)(x + 2)}$ has a relative maximum of $-\dfrac{4}{9}$ at $x = -\dfrac{1}{2}$.

$$\frac{d^2y}{dx^2} = \frac{6(x^2 + x + 1)}{(x - 1)^3 (x + 2)^3}$$

Possible inflection points:
$\dfrac{d^2y}{dx^2} \nexists$ when $x = 1$ or $x = -2$.
$\dfrac{d^2y}{dx^2} \neq 0$.

$y = \dfrac{1}{(x - 1)(x + 2)}$ is concave up on $(-\infty, -2) \cup (1, \infty)$.
$y = \dfrac{1}{(x - 1)(x + 2)}$ is concave down on $(-2, 1)$.
$y = \dfrac{1}{(x - 1)(x + 2)}$ has no inflection points.

6. Vertical asymptotes: $x = 1$ and $x = -1$
   Horizontal asymptotes: $y = -1$
   Oblique asymptotes: None

   $g'(x) = \dfrac{4x}{(x^2-1)^2}$

   Critical numbers:
   $g' \nexists$ when $x = 1$ or $x = -1$. (Neither are critical numbers but will be included in our analysis as usual.)
   g'(x) = 0 when x=0.

   Conclusions:
   g is increasing on $(0, 1) \cup (1, \infty)$.
   g is decreasing on $(-\infty, -1) \cup (-1, 0)$.

   g has a relative minimum of 1 at $x = 0$.

   $g''(x) = \dfrac{-4(3x^2+1)}{(x^2-1)^3}$

   Possible inflection points:
   $g'' \nexists$ when $x = 1$ or $x = -1$.
   $g''(x) \neq 0$.

   g is concave up on $(-1, 1)$.
   g is concave down on $(-\infty, -1) \cup (1, \infty)$.
   g has no inflection points.

7. Vertical asymptotes: $x = 0$ and $x = 1$ and $x = -1$
   Horizontal asymptotes: $y = 0$
   Oblique asymptotes: None

   $h'(x) = \dfrac{1-3x^2}{x^2(x^2-1)^2}$

   Critical numbers:
   $h' \nexists$ when $x = 1$ or $x = -1$ or $x = 0$. (None are critical numbers but will be included in our analysis as usual.)
   h'(x) = 0 when x=$\dfrac{\sqrt{3}}{3}$ or x=-$\dfrac{\sqrt{3}}{3}$.

   Conclusions:
   h is decreasing on $(-\infty, -1) \cup \left(-1, -\dfrac{\sqrt{3}}{3}\right) \cup \left(\dfrac{\sqrt{3}}{3}, 1\right) \cup (1, \infty)$
   h is increasing on $\left(-\dfrac{\sqrt{3}}{3}, 0\right) \cup \left(0, \dfrac{\sqrt{3}}{3}\right)$
   h has a relative minimum of $\dfrac{3\sqrt{3}}{2}$ at $x = -\dfrac{\sqrt{3}}{3}$.
   h has a relative maximum of $-\dfrac{3\sqrt{3}}{2}$ at $x = \dfrac{\sqrt{3}}{3}$.

   $h''(x) = \dfrac{6(6x^4-3x^2+1)}{x^3(x^2-1)^3}$

   $h'' \nexists$ when $x = 1$ or $x = -1$ or $x = 0$.
   $h''(x) \neq 0$.
   Conclusions:
   h is concave up on $(-1, 0) \cup (1, \infty)$.
   h is concave down on $(-\infty, -1) \cup (0, 1)$.
   h has no inflection points.

8. Vertical asymptotes: None
Horizontal asymptotes: None
Oblique asymptotes: None

$$\frac{dy}{dx} = \frac{3x+6}{2\sqrt{x+3}}$$

$\frac{dy}{dx} \nexists$ when $x = -3$
$\frac{dy}{dx} = 0$ when $x = -2$.

Conclusions:
$y = x\sqrt{x+3}$ is increasing on $(-2, \infty)$.
$y = x\sqrt{x+3}$ is decreasing on $(-3, 2)$.
$y = x\sqrt{x+3}$ has a relative minimum of -2 at $x = -2$

$$\frac{d^2y}{dx^2} = \frac{3x+12}{4\sqrt{(x+3)^3}}$$

$\frac{d^2y}{dx^2} \nexists$ when $x = -3$.
$\frac{d^2y}{dx^2} = 0$ when $x = -4$.

Conclusions:
$y = x\sqrt{x+3}$ is concave up everywhere in its domain. $\rightarrow$ $(-3, \infty)$.

9. Vertical asymptotes: None
Horizontal asymptotes: $y = 0$ (as $x \rightarrow \infty$ but none as $x \rightarrow -\infty$)
Oblique asymptotes: None

$$\frac{dy}{dx} = \frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}}$$

$\frac{dy}{dx} \exists \, \forall \, x$
$\frac{dy}{dx} \neq 0$

There are no critical numbers, therefore no relative extrema.

Conclusions:
$\frac{dy}{dx} < 0 \, \forall \, x \rightarrow y = \sqrt{x^2 = 1} - x$ is always decreasing.

$$\frac{d^2y}{dx^2} = \frac{1}{\sqrt{(x^2+1)^3}}$$

$\frac{d^2y}{dx^2} \exists \, \forall \, x$
$\frac{d^2y}{dx^2} \neq 0$

Conclusions:
$\frac{d^2y}{dx^2} > 0 \, \forall \, x \rightarrow y = \sqrt{x^2 = 1} - x$ is always concave up.

10. Vertical asymptotes: $x = 0$
Horizontal asymptotes: None
Oblique asymptotes: None

$$f'(x) = \frac{-1}{x^2\sqrt{1-x^2}}$$

$f' \nexists$ when $x = 0$ or $x = 1$ or $x = -1$
$f'(x) \neq 0$

Conclusions:
$f'(x) < 0 \; \forall \; x$ in $f \;\rightarrow\; f$ is always decreasing.

$$f''(x) = \frac{2-3x^2}{x^3\sqrt{(1-x^2)^3}}$$

$f'' \nexists$ when $x = 0$ or $x = 1$ or $x = -1$
$f''(x) = 0$ when $x = -\frac{\sqrt{6}}{3}$ or $x = \frac{\sqrt{6}}{3}$

Conclusions:
f is concave up on $\left(-1, -\frac{\sqrt{6}}{3}\right) \cup \left(0, \frac{\sqrt{6}}{3}\right)$.
f is concave down on $\left(-\frac{\sqrt{6}}{3}, 0\right) \cup \left(\frac{\sqrt{6}}{3}, 1\right)$.
f has inflection points at $\left(-\frac{\sqrt{6}}{3}, -\frac{\sqrt{2}}{2}\right)$ and $\left(\frac{\sqrt{6}}{3}, \frac{\sqrt{2}}{2}\right)$.

11. Vertical asymptotes: None
Horizontal asymptotes: None
Oblique asymptotes: None

$$\frac{dy}{dx} = x\sec^2 x + \tan x$$

$\frac{dy}{dx} \; \exists \; \forall \; x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\frac{dy}{dx} = 0$ when $x = 0$

Conclusions:
$y = x\tan x$ is decreasing on $\left(-\frac{\pi}{2}, 0\right)$
$y = x\tan x$ is increasing on $\left(0, \frac{\pi}{2}\right)$

$y = x\tan x$ has a relative minimum of 0 at $x = 0$.

$$\frac{d^2y}{dx^2} = 2\sec^2 x(x\tan x + 1)$$

$\frac{d^2y}{dx^2} \; \exists \; \forall \; x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\frac{d^2y}{dx^2} \neq 0$

Conclusions:
$2\sec^2 x(x\tan x + 1) > 0 \; \forall \; x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \;\rightarrow\; y = x\tan x$ is always concave up .

12. Vertical asymptotes: None
Horizontal asymptotes: None
Oblique asymptotes: None

$f'(x) = 2\cos 2x - 2\cos x$

$f' \exists \, \forall \, x \in (0, 2\pi)$
$f'(x) = 0$ when $x = \dfrac{2\pi}{3}$ or $x = \dfrac{4\pi}{3}$

Conclusions:
f is increasing on $\left(\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right)$.
f is decreasing on $\left(0, \dfrac{2\pi}{3}\right) \cup \left(\dfrac{4\pi}{3}, 2\pi\right)$.
f has a relative minimum of $-\dfrac{3\sqrt{3}}{2}$ at $x = \dfrac{2\pi}{3}$.
f has a relative maximum of $\dfrac{3\sqrt{3}}{2}$ at $x = \dfrac{4\pi}{3}$.

$f''(x) = -4\sin 2x + 2\sin x$

$f'' \exists \, \forall \, x \in (0, 2\pi)$
$f''(x) = 0$ when $x = 1.318$ or $x = \pi$ or $x = 4.965$

Conclusions:
f is concave down on $(0, 1.318) \cup (\pi, 4.965)$
f is concave up on $(1.318, \pi) \cup (4.965, 2\pi)$
f has inflection points at $(1.318, -1.452)$ and $(\pi, 0)$ and $(4.965, 1.453)$

13. Vertical asymptotes: x=-1
Horizontal asymptotes: y=1
Oblique asymptotes: None

$g'(x) = \dfrac{e^{-1/(x+1)}}{(x+1)^2}$

$g' \nexists$ when $x = -1$. Not a critical number.
$g'(x) \neq 0$

Conclusions:
g is increasing on $(-\infty, -1) \cup (-1, \infty)$.

g has no relative extrema.

$g''(x) = \dfrac{-(2x+1)e^{-1/(x+1)}}{(x+1)^4}$

$g'' \nexists$ when $x = -1$.
$g''(x) = 0$ when $x = 0$ or $x = -\dfrac{1}{2}$.

Conclusions:
g is concave down on $\left(-\dfrac{1}{2}, 0\right) \cup (0, \infty)$.
g is concave up on $(-\infty, -1) \cup \left(-1, -\dfrac{1}{2}\right)$.
g has an inflection point at $\left(-\dfrac{1}{2}, \dfrac{1}{e^2}\right)$

14. Vertical asymptotes: None
Horizontal asymptotes: y=1 and y=0
Oblique asymptotes: None

$\frac{dy}{dx} = \frac{e^x}{(e^x+1)^2}$

$\frac{dy}{dx} \exists \forall \, x$
$\frac{dy}{dx} \neq 0$

Conclusions:
No critical numbers so no relative extrema.
$y = \frac{1}{1+e^{-x}}$ is increasing $\forall \, x$

$\frac{d^2y}{dx^2} = \frac{e^x(1-e^x)}{(e^x+1)^2}$

$\frac{d^2y}{dx^2} \exists \forall \, x$
$\frac{d^2y}{dx^2} = 0$ when $x = 0$.

Conclusions:
$y = \frac{1}{1+e^{-x}}$ is concave up on $(-\infty, 0)$.
$y = \frac{1}{1+e^{-x}}$ is concave down on $(0, \infty)$.
$y = \frac{1}{1+e^{-x}}$ has an inflection point at $\left(0, \frac{1}{2}\right)$.

15. Vertical asymptotes: None
Horizontal asymptotes: None
Oblique asymptotes: None

$$\frac{dy}{dx} = \frac{2x}{1+x^2}$$

$\frac{dy}{dx} \exists \forall x$
$\frac{dy}{dx} = 0$ when $x = 0$.

Conclusions:
$y = \ln(1+x^2)$ is increasing on $(0, \infty)$.
$y = \ln(1+x^2)$ is decreasing on $(-\infty, 0)$.
$y = \ln(1+x^2)$ has a relative minimum of 0 at $x = 0$.

$$\frac{d^2y}{dx^2} = \frac{-2(x^2-1)}{(x^2+1)^2}$$

$\frac{d^2y}{dx^2} \exists \forall x$

$\frac{d^2y}{dx^2} = 0$ when $x = 1$ or $x = -1$.

Conclusions:
$y = \ln(1+x^2)$ is concave up on $(-1, 1)$.
$y = \ln(1+x^2)$ is concave down on $(-\infty, -1) \cup (1, \infty)$.
$y = \ln(1+x^2)$ has inflection points at $(-1, \ln 2)$ and $(1, \ln 2)$.

16. Vertical asymptotes: None
Horizontal asymptotes: None
Oblique asymptotes: None

$$h'(x) = 1 + \ln x$$

$h' \nexists$ when $x = 0$.
$h'(x) = 0$ when $x = \frac{1}{e}$.

Conclusions:
h is increasing on (e, ∞).
h is decreasing on $\left(0, \frac{1}{e}\right)$.
h has a relative minimum of $-\frac{1}{e}$ at $x = \frac{1}{e}$.

$$h''(x) = \frac{1}{x}$$

$h'' \nexists$ when $x = 0$.

$h''(x) \neq 0$.

Conclusions:
h is always concave up .
h has no inflection points .

17. Vertical asymptotes: x=0
Horizontal asymptotes: y=0
Oblique asymptotes: None

$$\frac{dy}{dx} = \frac{e^x(x-1)}{x^2}$$

$\dfrac{dy}{dx} \nexists$ when $x = 0$

$\dfrac{dy}{dx} = 0$ when $x = 1$.

Conclusions:
$y = \dfrac{e^x}{x}$ is increasing on $(1, \infty)$.
$y = \dfrac{e^x}{x}$ is decreasing on $(-\infty, 0) \cup (0, 1)$.
$y = \dfrac{e^x}{x}$ has a relative minimum of e at $x = 1$.

$$\frac{d^2y}{dx^2} = \frac{e^x(x^2 - 2x + 2)}{x^3}$$

$\dfrac{d^2y}{dx^2} \nexists$ at $x = 0$.

$\dfrac{d^2y}{dx^2} \neq 0$.

Conclusions:
$y = \dfrac{e^x}{x}$ is concave up on $(0, \infty)$.
$y = \dfrac{e^x}{x}$ is concave down on $(-\infty, 0)$.
$y = \dfrac{e^x}{x}$ has no inflection points .

18. Vertical asymptotes: x=1 and x=-1
Horizontal asymptotes: None
Oblique asymptotes: y=x

$$f'(x) = \frac{x^2(x^2-3)}{(x^2-1)^2}$$

$f' \nexists$ when $x = 1$ or $x = -1$.
$f'(x) = 0$ when $x = \sqrt{3}$ or $x = -\sqrt{3}$ or $x = 0$.

Conclusions:
f is increasing on $\left(-\infty, -\sqrt{3}\right) \cup \left(\sqrt{3}, \infty\right)$.

f is decreasing on $\left(-\sqrt{3}, -1\right) \cup (-1, 0) \cup \left(1, \sqrt{3}\right)$.

f has a relative maximum of $\frac{3^{3/2}}{2}$ at $x = -\sqrt{3}$.

f has a relative minimum of $\frac{3\sqrt{3}}{2}$ at $x = \sqrt{3}$.

$$f''(x) = \frac{2x^3 + 6x}{x^6 - 3x^4 + 3x^2 - 1}$$

$f'' \nexists$ when $x = 1$ or $x = -1$.
$f''(x) = 0$ when $x = 0$

Conclusions:
f is concave up on $(-1, 0) \cup (1, \infty)$
f is concave down on $(-\infty, -1) \cup (0, 1)$
f has an inflection point at $(0, 0)$

19. Vertical asymptotes: x=-$\frac{5}{2}$

Horizontal asymptotes: None
Oblique asymptotes: y=$\frac{1}{2}$x-$\frac{5}{4}$

$f'(x) = \frac{2x(x+5)}{(2x+5)^2}$

$f' \nexists$ when $x = -\frac{5}{2}$.
$f'(x) = 0$ when $x = 0$ or $x = -5$.

Conclusions:
f is increasing on $(-\infty, -5) \cup (0, \infty)$.

f is decreasing on $\left(-5, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, 0\right)$.

f has a relative maximum of -5 at $x = -5$.

f has a relative minimum of 0 at $x = 0$.

$f''(x) = \frac{50}{(2x+5)^3}$

$f'' \nexists$ when $x = -\frac{5}{2}$.
$f''(x) \neq 0$.

Conclusions:
f is concave up on $\left(-\frac{5}{2}, \infty\right)$

f is concave down on $\left(-\infty, -\frac{5}{2}\right)$

f has no inflection points .

20. Vertical asymptotes: None
Horizontal asymptotes: None
Oblique asymptotes: None

$f'(x) = \frac{1}{2} - \cos x$

$f' \; \exists \; \forall \; x \in (0, 3\pi)$.
$f'(x) = 0$ when $x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$ or $x = \frac{7\pi}{3}$.

Conclusions:
f is increasing on $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right) \cup \left(\frac{7\pi}{3}, \infty\right)$.

f is decreasing on $\left(-\infty, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, \frac{7\pi}{3}\right)$.

f has a relative minimum of $\frac{\pi - 3\sqrt{3}}{6}$ at $x = \frac{\pi}{3}$.

f has a relative minimum of $\frac{7\pi - 3\sqrt{3}}{6}$ at $x = \frac{7\pi}{3}$.

f has a relative maximum of $\frac{5\pi + 3\sqrt{3}}{6}$ at $x = \frac{5\pi}{3}$.

$f''(x) = \sin x$

$f'' \; \exists \; \forall \; x \in (0, 3\pi)$.
$f''(x) = 0$ when $x = \pi$ or $x = 2\pi$.

Conclusions:
f is concave up on $(0, \pi) \cup (2\pi, 3\pi)$

f is concave down on $(\pi, 2\pi)$

f has an inflection point at $(2\pi, \pi)$.