1. $f'(x) = 3x^2 - 1$

   Critical numbers:
   $f' \exists \forall x$.
   $f'(x) = 0$ when $3x^2 - 1 = 0 \rightarrow x = \dfrac{\sqrt{3}}{3}$ or $x = -\dfrac{\sqrt{3}}{3}$.

   Second Derivative Test:

   $f''\left(-\dfrac{\sqrt{3}}{3}\right) = -2\sqrt{3} < 0 \rightarrow f$ is concave down $\rightarrow f$ has a relative maximum of $\dfrac{2\sqrt{3}}{9}$ at $x = -\dfrac{\sqrt{3}}{3}$.
   $f''\left(\dfrac{\sqrt{3}}{3}\right) = 2\sqrt{3} > 0 \rightarrow f$ is concave up $\rightarrow f$ has a relative minimum of $-\dfrac{2\sqrt{3}}{9}$ at $x = \dfrac{\sqrt{3}}{3}$.

2. $f'(x) = 6x^2 + 10x - 4$

   Critical numbers:
   $f' \exists \forall x$.
   $f'(x) = 0$ when $6x^2 + 10x - 4 = 0 \rightarrow x = -2$ or $x = \dfrac{1}{3}$.

   Second derivative test:

   $f''(-2) = -14 < 0 \rightarrow f$ is concave down $\rightarrow f$ has a relative maximum of 12 at $x = -2$.
   $f''\left(\dfrac{1}{3}\right) = 14 > 0 \rightarrow f$ is concave up $\rightarrow f$ has a relative minimum of $-\dfrac{19}{27}$ at $x = \dfrac{1}{3}$.

3. $f'(x) = 3x^2 - 1$
   $f''(x) = 6x$

   Possible inflection points:
   $f'' \exists \forall x$
   $f''(x) = 0$ when $x = 0$

   Analysis:
   $f''(x) < 0$ on $(-\infty, 0) \rightarrow f$ is concave down on $(-\infty, 0)$.
   $f''(x) > 0$ on $(0, \infty) \rightarrow f$ is concave up on $(0, \infty)$.
   Since $f''(x) > 0$ on $(0, \infty)$ and $f''(x) < 0$ on $(-\infty, 0)$ and $f(0) = 0$, f has an inflection point at $(0, 0)$.

4. $f'(x) = 4x^3 - 12x$
   $f''(x) = 12x^2 - 12$

   Possible inflection points:
   $f'' \exists \forall x$
   $f''(x) = 0$ when $12x^2 - 12 = 0 \rightarrow x = 1$ or $x = -1$.

   Analysis:
   Since $f''(x) > 0$ on $(-\infty, -1) \cup (1, \infty) \rightarrow f$ is concave up on $(-\infty, -1) \cup (1, \infty)$.
   Since $f''(x) < 0$ on $(-1, 1) \rightarrow f$ is concave down on $(-1, 1)$.

   Since $f''(x) > 0$ on $(-\infty, -1)$ and $f''(x) < 0$ on $(-1, 1)$ and $f(-1) = -5$, f has an inflection point at $(-1, -5)$.
   Since $f''(x) < 0$ on $(-1, 1)$ and $f''(x) > 0$ on $(1, \infty)$ and $f(1) = -5$, f has an inflection point at $(1, -5)$.

5. $f'(x) = 15x^4 - 15x^2$
   $f''(x) = 60x^3 - 30x$

   Possible inflection points:
   $f'' \exists \forall x$
   $f''(x) = 0$ when $60x^3 - 30x = 0 \rightarrow x = 0$ or $x = .707$ or $x = -.707$.

   Analysis:
   Since $f''(x) > 0$ on $(-.707, 0) \cup (.707, \infty) \rightarrow f$ is concave up on $(-.707, 0) \cup (.707, \infty)$.
   Since $f''(x) < 0$ on $(-\infty, -.707) \cup (0, .707) \rightarrow f$ is concave down on $(-\infty, -.707) \cup (0, .707)$.

   Since $f''(x) < 0$ on $(-\infty, -.707)$ and $f''(x) > 0$ on $(-.707, 0)$ and $f(-.707) = 4.237$,
   f has an inflection point at $(-.707, 4.237)$.
   Since $f''(x) > 0$ on $(-.707, 0)$ and $f''(x) < 0$ on $(0, .707)$ and $f(0) = 3$,
   f has an inflection point at $(0, 3)$.
   Since $f''(x) < 0$ on $(0, .707)$ and $f''(x) > 0$ on $(.707, \infty)$ and $f(.707) = 1.763$,
   f has an inflection point at $(.707, 1.763)$.

6. $P'(x) = \dfrac{2x^2 + 1}{\sqrt{x^2 + 1}}$
   $P''(x) = \dfrac{2x^3 + 3x}{\sqrt{(x^2 + 1)^3}}$
   Possible inflection points:
   $P'' \exists \forall x$
   $P''(x) = 0$ when $2x^3 + 3x = 0 \rightarrow x = 0$

   Analysis:
   Since $P''(x) > 0$ on $(0, \infty) \rightarrow P$ is concave up on $(0, \infty)$.
   Since $P''(x) < 0$ on $(-\infty, 0) \rightarrow P$ is concave down on $(-\infty, 0)$.

   Since $P''(x) < 0$ on $(-\infty, 0)$ and $P''(x) > 0$ on $(0, \infty)$ and $P(0) = 0$, P has an inflection point at $(0, 0)$.

7. $f'(x) = \dfrac{x + 1}{x^{2/3}(x + 3)^{1/3}}$
   $f''(x) = \dfrac{-2}{x^{5/3}(x + 3)^{4/3}}$
   Possible inflection points:
   $f'' \nexists$ when $x^{5/3}(x + 3)^{4/3} = 0 \rightarrow x = 0$ or $x = -3$
   $f''(x)$ never zero

   Analysis:
   Since $f''(x) > 0$ on $(-\infty, -3) \cup (-3, 0) \rightarrow f$ is concave up on $(-\infty, -3) \cup (-3, 0)$.
   Since $f''(x) < 0$ on $(0, \infty) \rightarrow f$ is concave down on $(0, \infty)$

   Since $f''(x) > 0$ on $(-3, 0)$ and $f''(x) < 0$ on $(0, \infty)$ and $f(0) = 0$, f has an inflection point at $(0, 0)$.

8. $h'(\theta) = 2\sin\theta\cos\theta$
$h''(\theta) = 4\cos^2\theta - 2$

Possible inflection points:
$h''(\theta) \ \exists \ \forall \ x \in [0, 2\pi]$
$h''(\theta) = 0$ when $\theta = \dfrac{\pi}{4}$ or $\theta = \dfrac{3\pi}{4}$ or $\theta = \dfrac{5\pi}{4}$ or $\theta = \dfrac{7\pi}{4}$

Analysis:
Since $h''(\theta) > 0$ on $\left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right) \cup \left(\dfrac{7\pi}{4}, 2\pi\right) \ \rightarrow \ h$ is concave up on $\left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right) \cup \left(\dfrac{7\pi}{4}, 2\pi\right)$.
Since $h''(\theta) < 0$ on $\left(\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right) \cup \left(\dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right) \ \rightarrow \ h$ is concave down on $\left(\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right) \cup \left(\dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right)$.

Since $h''(\theta) > 0$ on $\left(0, \dfrac{\pi}{4}\right)$ and $h''(\theta) < 0$ on $\left(\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right)$ and $h\left(\dfrac{\pi}{4}\right) = \dfrac{1}{2}$, h has an inflection point at $\left(\dfrac{\pi}{4}, \dfrac{1}{2}\right)$.
Since $h''(\theta) < 0$ on $\left(\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right)$ and $h''(\theta) > 0$ on $\left(\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right)$ and $h\left(\dfrac{3\pi}{4}\right) = \dfrac{1}{2}$, h has an inflection point at $\left(\dfrac{3\pi}{4}, \dfrac{1}{2}\right)$.
Since $h''(\theta) > 0$ on $\left(\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right)$ and $h''(\theta) < 0$ on $\left(\dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right)$ and $h\left(\dfrac{5\pi}{4}\right) = \dfrac{1}{2}$, h has an inflection point at $\left(\dfrac{5\pi}{4}, \dfrac{1}{2}\right)$.
Since $h''(\theta) < 0$ on $\left(\dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right)$ and $h''(\theta) > 0$ on $\left(\dfrac{7\pi}{4}, 2\pi\right)$ and $h\left(\dfrac{7\pi}{4}\right) = \dfrac{1}{2}$, h has an inflection point at $\left(\dfrac{7\pi}{4}, \dfrac{1}{2}\right)$.

9. $\dfrac{dy}{dx} = -4x^3 - 6x^2 + 12x$
$\dfrac{d^2y}{dx^2} = -12x^2 - 12x + 12$

Possible inflection points:
$\dfrac{d^2y}{dx^2} \ \exists \ \forall \ x$
$\dfrac{d^2y}{dx^2} = 0$ when $x = .618$ or $x = -1.618$

Analysis:
Since $\dfrac{d^2y}{dx^2} < 0$ on $(-\infty, -1.618) \cup (.618, \infty) \ \rightarrow \ y = 6x^2 - 2x^3 - x^4$ is concave down on $(-\infty, -1.618) \cup (.618, \infty)$.
Since $\dfrac{d^2y}{dx^2} > 0$ on $(-1.618, .618) \ \rightarrow \ y = 6x^2 - 2x^3 - x^4$ is concave up on $(-1.618, .618)$.

Since $\dfrac{d^2y}{dx^2} < 0$ on $(-\infty, -1.618)$ and $\dfrac{d^2y}{dx^2} > 0$ on $(-1.618, .618)$ and $y(-1.618) = 17.326$,
$y = 6x^2 - 2x^3 - x^4$ has an inflection point at $(-1.618, 17.326)$

Since $\dfrac{d^2y}{dx^2} > 0$ on $(-1.618, .618)$ and $\dfrac{d^2y}{dx^2} < 0$ on $(.618, \infty)$ and $y(.618) = 1.674$,
$y = 6x^2 - 2x^3 - x^4$ has an inflection point at $(.618, 1.674)$

10. $\dfrac{dy}{dx} = \dfrac{1-x}{(x+1)^3}$

$\dfrac{d^2y}{dx^2} = \dfrac{2(x-2)}{(x+1)^4}$

Possible inflection points:

$\dfrac{d^2y}{dx^2} \nexists$ when $x = -1$.

$\dfrac{d^2y}{dx^2} = 0$ when $x = 2$

Analysis:

Since $\dfrac{d^2y}{dx^2} < 0$ on $(-\infty, -1) \cup (-1, 2) \ \rightarrow \ y = \dfrac{x}{(1+x)^2}$ is concave down on $(-\infty, -1) \cup (-1, 2)$.

Since $\dfrac{d^2y}{dx^2} > 0$ on $(2, \infty); \rightarrow \ y = \dfrac{x}{(1+x)^2}$ is concave up on $(2, \infty)$.

Since $\dfrac{d^2y}{dx^2} < 0$ on $(-1, 2)$ and $\dfrac{d^2y}{dx^2} > 0$ on $(2, \infty)$ and $y(2) = \dfrac{2}{9}, y = \dfrac{x}{(1+x)^2}$ has an inflection point at $\left(2, \dfrac{2}{9}\right)$.

11. $\dfrac{dy}{dx} = e^x(x+1)$

$\dfrac{d^2y}{dx^2} = e^x(x+2)$

Possible inflection points:

$\dfrac{d^2y}{dx^2} \exists \ \forall \ x$

$\dfrac{d^2y}{dx^2} = 0$ when $x = -2$

Analysis:

Since $\dfrac{d^2y}{dx^2} > 0$ on $(-2, \infty) \ \rightarrow \ y = xe^x$ is concave up on $(-2, \infty)$.

Since $\dfrac{d^2y}{dx^2} < 0$ on $(-\infty, -2) \ \rightarrow \ y = xe^x$ is concave down on $(-\infty, -2)$.

Since $\dfrac{d^2y}{dx^2} > 0$ on $(-2, \infty)$ and $\dfrac{d^2y}{dx^2} < 0$ on $(-\infty, -2)$, and $y(-2) = -\dfrac{2}{e^2}, y = xe^x$ has an inflection point at $\left(-2, -\dfrac{2}{e^2}\right)$

12. $f'(x) = \dfrac{2 - \ln x}{2\sqrt{x^3}}$

$f''(x) = \dfrac{-8 + 3\ln x}{4\sqrt{x^5}}$

Possible inflection points:

$f' \nexists$ when $x = 0$

$f''(x) = 0$ when $-8 + 3\ln x = 0 \ \rightarrow \ x = e^{8/3}$

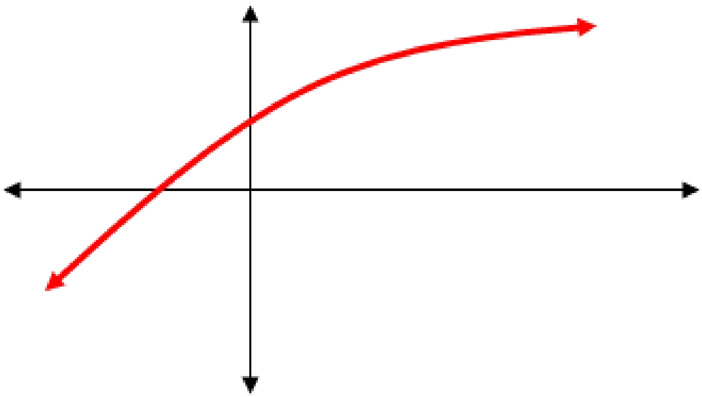
Analysis:

Since $f''(x) > 0$ on $(e^{8/3}, \infty) \ \rightarrow \ f$ is concave up on $(e^{8/3}, \infty)$.

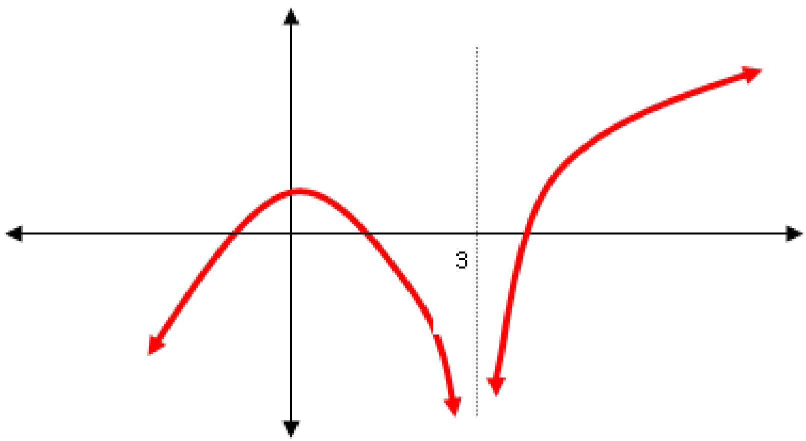
Since $f''(x) < 0$ on $(0, e^{8/3}) \ \rightarrow \ f$ is concave down on $(0, e^{8/3})$.

Since $f''(x) < 0$ on $(0, e^{8/3})$ and $f''(x) > 0$ on $(e^{8/3}, \infty)$, and $f(e^{8/3}) = \dfrac{8}{3e^{4/3}}, f$ has an inflection point at $\left(e^{8/3}, \dfrac{8}{3e^{4/3}}\right)$

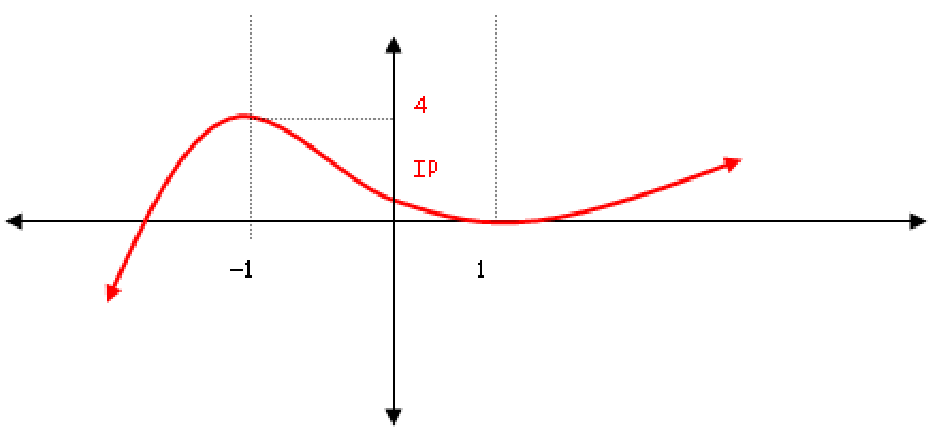
13. Your sketch should look similar to this one:



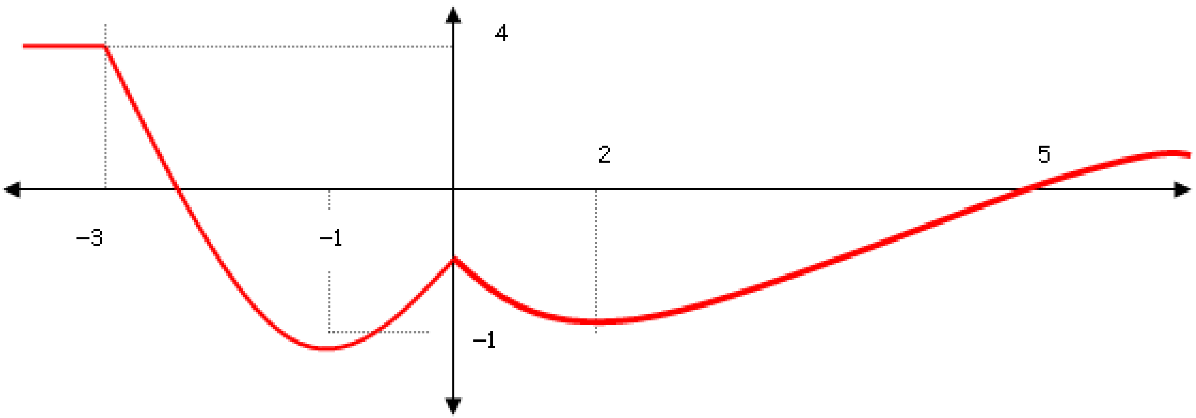
14. Your sketch should look similar to this one:



15. Your sketch should look similar to this one:



16. Your sketch should look similar to this one:



17. Your sketch should look similar to this one: