1. False.

   $f'(x) = -\sin x$

   - $f' \exists \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
   - $f'(x) = 0$ when $x = 0$

   Since $f'(x) > 0$ on $\left(-\frac{\pi}{2}, 0\right)$ and $f'(x) < 0$ on $\left(0, \frac{\pi}{2}\right)$ f is not one-to-one.

2. True.

   e^x can never be zero.

3. False.

   $\ln 10$ is a constant and its derivative is zero.

4. False.

   $\cos^{-1} x$ is the inverse of the cosine function, not $\frac{1}{\cos x}$.

5. True.

   $\frac{d}{dx} \log_8 x = \frac{1}{x \ln 8} = \frac{1}{x \ln 2^3} = \frac{1}{(3 \ln 2)x}$

6. $e^x = 5$

   $\ln e^x = \ln 5$

   $x = \ln 5$

7. $\log_{10} e^x = 1$

   $10^1 = e^x$

   $\ln 10 = \ln e^x$

   $x = \ln 10$

8. $\ln x^\pi = 2$

   $e^2 = x^\pi$

   $\ln e^2 = \ln x^\pi$

   $2 = \pi \ln x$

   $\frac{2}{\pi} = \ln x$

   $x = e^{2/\pi}$

9. $x = \tan^{-1} 4$

10. $$\frac{dy}{dx} = \frac{2x-1}{(x^2-x)\ln 10}$$

or

$$\frac{dy}{dx} = \frac{(2x-1)\log_{10} e}{(x^2-x)}$$

11. $$y = \left(\sqrt{2}\right)^x$$

$$\ln y = \ln\left(\sqrt{2}\right)^x$$

$$\ln y = x \ln\sqrt{2}$$

$$\frac{1}{y}\,\frac{dy}{dx} = \ln\sqrt{2}$$

$$\frac{dy}{dx} = y\ln\sqrt{2}$$

$$\frac{dy}{dx} = \left(\sqrt{2}\right)^x\ \ln\sqrt{2}$$

12. $$\begin{aligned} f'(x) &= (e^{cx})(c\cos x + \sin x) + (c\sin x - \cos x)ce^{cx} \\ &= e^{cx}(c\cos x + \sin x + c^2\sin x - c\cos x) \\ &= e^{cx}(\sin x + c^2\sin x) \\ &= e^{cx}(\sin x)(1+c^2) \end{aligned}$$

13. $$\begin{aligned} g'(x) &= \frac{2\sec x\sec x\tan x}{\sec^2 x} \\ &= 2\tan x \end{aligned}$$

14. $$\begin{aligned} y &= xe^{x^{-1}} \\ \frac{dy}{dx} &= (x)\left(-e^{x^{-1}}x^{-2}\right) + \left(e^{x^{-1}}\right)(1) \\ &= \frac{xe^{x^{-1}} - e^{x^{-1}}}{x} \\ &= \frac{e^{1/x}(x-1)}{x} \end{aligned}$$

15. $$\begin{aligned} f'(x) &= \left(7^{\sqrt{2x}}\right)\left[\frac{1}{2}(2x)^{-1/2}(2)\right]\ln 7 \\ &= \frac{7^{\sqrt{2x}}\ln 7}{\sqrt{2x}} \end{aligned}$$

16.
$$y = e^{e^x}$$
$$\ln y = \ln e^{e^x}$$
$$\ln y = e^x \ln e$$
$$\frac{1}{y}\,\frac{dy}{dx} = e^x$$
$$\frac{dy}{dx} = ye^x$$
$$\frac{dy}{dx} = e^{e^x} e^x$$
$$\frac{dy}{dx} = e^{e^x + x}$$

17.
$$\begin{aligned} f(x) &= \ln x^{-1} + (\ln x)^{-1} \\ &= -\ln x + (\ln x)^{-1} \\ f'(x) &= -\frac{1}{x} - (\ln x)^{-2}\,\frac{1}{x} \\ &= -\frac{1}{x} - \frac{1}{x \ln^2 x} \\ &= \frac{-\ln^2 x - 1}{x \ln^2 x} \end{aligned}$$

18.
$$\begin{aligned} \frac{dy}{dx} &= \frac{\dfrac{(x+1)(1) - (x-1)(1)}{(x+1)^2}}{\sqrt{1 - \left(\dfrac{x-1}{x+1}\right)^2}} \\ &= \frac{1}{|x+1|\,\sqrt{x}} \end{aligned}$$

19. Point

Given as $(0, \ln 2)$

Slope

$$\frac{dy}{dx} = \frac{e^x + 2e^{2x}}{e^x + e^{2x}} \longrightarrow \left.\frac{dy}{dx}\right|_{(0,\ln 2)} = \frac{3}{2}$$

Equation of tangent

$$y - \ln 2 = \frac{3}{2}(x - 0)$$

20. Point

    Given as (e, e)

    Slope

    $$\frac{dy}{dx} = x\,\frac{1}{x} + \ln x \;\longrightarrow\; \left.\frac{dy}{dx}\right|_{(e,e)} = 2$$

    Equation of tangent

    $y - e = 2(x - e)$

21. $$\begin{aligned}\frac{dy}{dx} &= [2\ln(x+4)]\;\frac{1}{x+4}\\ &= \frac{2\ln(x+4)}{x+4}\end{aligned}$$

    Now, the tangent will be horizontal when $\dfrac{dy}{dx} = 0.$

    $\dfrac{2\ln(x+4)}{x+4} = 0$ when $2\ln(x+4) = 0 \;\longrightarrow\; x = -3$

    When $x = -3 \;\longrightarrow\; y = 0$

    Therefore, at the point $(-3, 0)$ the tangent will be horizontal.

22. The slope of the tangent from the given line is $\dfrac{1}{4}$.

    The slope of the tangent from the derivative is $\dfrac{dy}{dx} = e^x$.

    To find the point, we will set the two expressions for the slope of the tangent equal.

    $e^x = \dfrac{1}{4} \;\longrightarrow\; x = -\ln 4 \;\longrightarrow\; y = \dfrac{1}{4}$

    Therefore the point is $\left(-\ln 4, \dfrac{1}{4}\right)$ and the equation of the tangent is $y - \dfrac{1}{4} = \dfrac{1}{4}(x + \ln 4)$.

23. To solve this problem we need two expressions for the slope of the tangent.

    The slope of the tangent from the derivative is $\dfrac{dy}{dx} = e^x$.

    Since any point on the curve can be written (x, e^x) we can find another expression for the slope of the tangent by writing the slope of the line through (x, e^x) and $(0, 0)$ . . . since the required tangent also passes through the origin.

    $m = \dfrac{e^x - 0}{x - 0} = \dfrac{e^x}{x}$

    We can now find points by setting these two expressions equal.

    $e^x = \dfrac{e^x}{x} \;\longrightarrow\; x = 1 \;\longrightarrow\; y = e.$

    Therefore the point is $(1, e)$ which means the slope of the tangent is e (from the value of the derivative at $x = 1$) and the equation of the tangent is $y - e = e(x - 1)$.

24. Bacteria in 3 hours

    $y = y_o e^{kt}$

    For $y = 9000$ and $y_o = 1000$ and $t = 2 \longrightarrow k = \ln 3$.

    Now, $y = 1000e^{t(\ln 3)}$

    For $t = 3 \longrightarrow y = 27000$

    Therefore in 3 hours there will be 27000 bacteria.

    Doubling time

    Using the doubling time formula $t = \dfrac{\ln 2}{k}$ we get $t = \dfrac{\ln 2}{\ln 3} \approx .631$.

    Therefore the doubling time is about .631 hours.

25. $f'(x) = g'(e^x)\ e^x$

26. $f'(x) = g'(x)\ e^{g(x)}$

27. $f'(x) = g'(\ln x)\ \dfrac{1}{x} = \dfrac{g'(\ln x)}{x}$

28. $f'(x) = (x)\left(e^{g(\sqrt{x})}\right)(g'(\sqrt{x}))\left(\dfrac{1}{2\sqrt{x}}\right) + e^{g(\sqrt{x})}$

29. $f'(x) = \dfrac{1}{g(e^x)} \cdot g'(e^x)\ e^x = \dfrac{g'(e^x)\ e^x}{g(e^x)}$

30. $f'(x) = -\dfrac{10}{(3x-1)^2}$

    f has an inverse because $f'(x) \leq 0\ \forall\ x$ in f.

31. $f'(x) = \dfrac{1}{2\sqrt{x-4}}$

    f has an inverse because $f'(x) \geq 0\ \forall\ x$ in f.

32. $f'(x) = 5x^4 + 3x^2 + 1$

    f has an inverse because $f'(x) \geq 0\ \forall\ x$ in f.

33. Let $y = f(x)$

$$y = \frac{x+3}{x-6}$$

$$xy - 6y = x + 3$$

$$x = \frac{6y+3}{y-1}$$

$$f^{-1}(y) = \frac{6y+3}{y-1}$$

$$f^{-1}(x) = \frac{6x+3}{x-1}$$

34. Let $y = f(x)$

$$y = e^{3x-4}$$

$$\ln y = 3x - 4$$

$$x = \frac{4 + \ln y}{3}$$

$$f^{-1}(y) = \frac{4 + \ln y}{3}$$

$$f^{-1}(x) = \frac{4 + \ln x}{3}$$