1. Horizontal line test.
2. $f'(x) = 7$

   f has an inverse because $f'(x) \geq 0 \; \forall \; x \;$ in $\; f$.
3. $g'(x) = \dfrac{1}{2\sqrt{x}}$

   f has an inverse because $f'(x) \geq 0 \; \forall \; x \;$ in $\; f$.
4. $h'(x) = 4x^3$

   f has no inverse because $h'(x) < 0 \;$ on $\; (-\infty, 0) \;$ and $\; h'(x) > 0 \;$ on $\; (0, \infty)$.
5. $f'(x) = \dfrac{17}{(5-2x)^2}$

   f has an inverse because $f'(x) \geq 0 \; \forall \; x \;$ in $\; f$.
6. $f'(x) = \dfrac{5}{2\sqrt{2+5x}}$

   f has an inverse because $f'(x) \geq 0 \; \forall \; x \;$ in $\; f$.
7. Finding the inverse

   Let $y = f(x)$

   $$y = 2x + 1$$

   $$x = \frac{y-1}{2}$$

   $$f^{-1}(y) = \frac{y-1}{2}$$

   $$f^{-1}(x) = \frac{x-1}{2}$$

   Derivative of $f^{-1}(x)$

   $$\left(f^{-1}\right)'(x) = \frac{1}{2}$$

   $$\left(f^{-1}\right)'(3) = \frac{1}{2}$$

   Using the theorem

   $2c + 1 = 3 \;\longrightarrow\; c = 1 \;\longrightarrow\; (1, 3) \;$ is on $\; f$.

   $f'(x) = 2 \;\longrightarrow\; f'(1) = 2$

   Since $(1, 3)$ is on f, $\left(f^{-1}\right)'(3) = \dfrac{1}{f'(1)} = \dfrac{1}{2}$

8. Finding the inverse

Let $y = f(x)$

$$y = x^3$$

$$x = y^{1/3}$$

$$f^{-1}(y) = y^{1/3}$$

$$f^{-1}(x) = x^{1/3}$$

Derivative of $f^{-1}(x)$

$$\left(f^{-1}\right)'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

$$\left(f^{-1}\right)'(8) = \frac{1}{12}$$

Using the theorem

$c^3 = 8 \longrightarrow c = 2 \longrightarrow (2,8)$ is on f.

$f'(x) = 3x^2 \longrightarrow f'(2) = 12$

Since $(2,8)$ is on f, $\left(f^{-1}\right)'(8) = \dfrac{1}{f'(2)} = \dfrac{1}{12}$

9. Finding the inverse

Let $y = f(x)$

$$y = 9 - x^2$$

$$x = \sqrt{9-y}$$

$$f^{-1}(y) = \sqrt{9-y}$$

$$f^{-1}(x) = \sqrt{9-x}$$

Derivative of $f^{-1}(x)$

$$\left(f^{-1}\right)'(x) = -\frac{1}{2\sqrt{9-x}}$$

$$\left(f^{-1}\right)'(8) = -\frac{1}{2}$$

Using the theorem

$9 - c^2 = 8 \longrightarrow c = 1$ or $c = -1$ but -1 not in $f \longrightarrow (1,8)$ is on f.

$f'(x) = -2x \longrightarrow f'(1) = -2$

Since $(1,8)$ is on f, $\left(f^{-1}\right)'(8) = \dfrac{1}{f'(1)} = -\dfrac{1}{2}$

10. $c^3+c+1=1 \longrightarrow c=0 \longrightarrow (0,1)$ is on f.

$f'(x)=3x^2+1 \longrightarrow f'(0)=1$

Since $(0,1)$ is on f, $\left(f^{-1}\right)'(1)=\dfrac{1}{f'(0)}=1$

11. $c^5-c^3+2c=2 \longrightarrow c=1 \longrightarrow (1,2)$ is on f.

$f'(x)=5x^4-3x^2+2 \longrightarrow f'(1)=4$

Since $(1,2)$ is on f, $\left(f^{-1}\right)'(2)=\dfrac{1}{f'(1)}=\dfrac{1}{4}$

12. $\sqrt{c^3+c^2+c+1}=2 \longrightarrow c=1 \longrightarrow (1,2)$ is on f.

$f'(x)=\dfrac{3x^2+2x+1}{2\sqrt{x^3+x^2+x+1}} \longrightarrow f'(1)=\dfrac{3}{2}$

Since $(1,2)$ is on f, $\left(f^{-1}\right)'(2)=\dfrac{1}{f'(1)}=\dfrac{2}{3}$

13. $\dfrac{1+3c}{5-2c}=2 \longrightarrow c=\dfrac{9}{7} \longrightarrow \left(\dfrac{9}{7},2\right)$ is on f.

$f'(x)=\dfrac{17}{(5-2x)^2} \longrightarrow f'\left(\dfrac{9}{7}\right)=\dfrac{49}{17}$

Since $\left(\dfrac{9}{7},2\right)$ is on f, $\left(f^{-1}\right)'(2)=\dfrac{1}{f'\left(\dfrac{9}{7}\right)}=\dfrac{17}{49}$